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Maximization of Revenue by using the Concept of Differentiation

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Abstract

The aim of this presentation is to explore the knowledge of revenue maximization with the help of second derivative test, is a part of classical optimization techniques applied to the general problems occurred in real time.

Keywords: Objective function, Revenue function, Cost function, Price function, Profit function.

I. INTRODUCTION

In calculus when y is a function of x, the derivative of y w.r.to x measures the rate of change of y w.r.to x. In economics and commerce we come across many such variables where one variable is a function of the other. Marginal analysis in economics and commerce is the most direct application of differential calculus. Differential calculus helps in finding the maximum profit and minimum cost etc. Here we discuss some problems related to maximization of revenue.

First we discuss some basic definitions related the problems.

1.1 Cost function: The cost C of producing and marketing x units of a product depends upon the number of units of x. So the function relating C and x is called cost function and is denoted by C(x).

For producing x units of the product the total cost consists of two parts called fixed cost and variable cost. The fixed cost consists of all types of costs which do not change with the level of production where as the variable cost is the sum of all costs that are depend on the level of production.

1.2 Demand function: An equation that relates price per unit and quantity demanded at that price is called a demand function.

1.3 Revenue function: If ‘x’ is the number of units of a certain product sold at a rate of Rs.p per unit, then the amount derived from the sale of x units of a product is called the total revenue.

If R represents the total revenue from x units of the product then R(x) = x.p or x.p(x)

1.4 Profit function: The profit is calculated by subtracting the total cost from the total revenue obtained by selling x units of the product. We generally denoted this function by P(x). Thus the profit function P(x) = R(x)-C(x).

1.5 Break-Even Point: Breakeven point is that the value of x (the number of units of the products sold) for which there is no profit or loss.

At the breakeven point P(x) =0, which shows at this value of x, R(x) =C(x).

1.6 Classical Optimization Technique (second derivative test): This technique tell us that when we are interested to find the level of output for which the total revenue is maximum we have to find the derivate of R(x) and equate with zero so that we get the value of x for which The second derivate is less than zero. Similarly we can find the value of x for maximum profit.

II. Applications of Second Derivative Test

2.1 Maximization of revenue to a transportation problem

TSRTC (Telangana State Road Transport Corporation) runs the buses between the state capitals of Telangana (Hyderabad) and Andhra Pradesh (Amravati) with a fare of Rs.300. The capacity for passengers in one bus is 40. On this route the average number of passengers travel are 20. Because of having competition with the private transport system, if the fare is reduced by Rs.10 may attract two more passengers for each bus. How should the management of TSRTC set the fare to maximize their revenue?

Solution: Let x be the number of times the fare is reduced by Rs.10 and R(x) be the revenue function. Then Price =300-(10) x rupees, Quantity= number of passengers = 20+x (2).

Since $R(x) = \text{Quantity} \times \text{Price}$
$= (20+2x)(300-10x)$
where $0 \leq x \leq 10$ because the total number of passengers can travel in a bus is 40.

$$R(x) = 6000+400x-20x^2 \text{ ------- } (1)$$

Differentiate equation (1) with respect to 'x' on both sides, then we get

$$R^1(x) = 400-40x \text{ ---------------- } (2)$$

To get the critical value for 'x', put $R^1(x) = 0$, then we get x=10.

Differentiate equation (2), $R^{11}(x) = -40 < 0$, from the second derivative test R(x) has absolute maximum value at x=10.

Therefore the best fare to maximize the revenue is 300-10(10) = Rs. 200, with passengers 20+2x=20+2(10) = 40 and the total revenue R(x) = 40×Rs.200= Rs.8000. Otherwise the average income is Rs.6000 (Rs.300× 20 persons).

2.2 Utilization of farm land for plantation to maximize the total yield

A farmer estimates that if 60 mango trees are planted in an acre, the average yield per tree will be 400 mangos. The average yield will decrease by 4 mangos per tree for each addition tree planted in an acre. How many trees should the farmer plant to maximize the total yield? If he has 20 acres of land and will get the profit Rs.2 on each mango then find his profit?

Solution: Let 'x' be the number of additional trees and 'y' be the total yield. Then y(x) can be defined as

$$y(x) = \text{number of trees} \times \text{yield per tree}$$
$$= (60 \text{ trees} + x \text{ trees}) \times (400 \text{ mango's} - 4x \text{ mango's})$$
$$= (60+x)(400-4x)$$
$$= 24000+160x-4x^2 \text{ ----------- } (1)$$

Differentiate both sides with respect to 'x', we get

$$y^1(x) = dy/dx = 160-8x \text{ -------------- } (2)$$

To get the critical value put $y^1(x) = 0$, so that we get x=20.

Now differentiate equation (2) w.r.to x we get $y^{11}(x) = -8 < 0$. This shows y(x) attains the maximum value at x=20.
Therefore the farmer can plant 80 trees so that the maximum yield $[y(x)] = 80\times(400-80) =$ 25,600 mango's.

2.3 Estimation of time for optimal production and maximization of revenue

Cotton farmers can get Rs.20,000 per ton of their cotton on 26th November and after that the price will drops by Rs.200 per ton for each extra day. A farmer has 80 tons of cotton in the field and estimates that it will increase at the rate one ton per day. When the farmer should sell the cotton to maximize his revenue?

Solution: Let 'x' be the number of extra day after 26th November and 'R' be the revenue then R(x) = Quantity × Price.

Here the Quantity = number of tons + x
= 80 + x
Price = 20,000- x (200)
= 200 (200-x)
Therefore revenue R(x) = (80+x). 200 (200-x)

$$= 16,00,000-4,000x-2000x^2$$

Now $R^1(x) = 4000-400x$, put $R^1(x) = 0$ we get x=10.
Finding the second derivative at x=10 we have $R^{11}(x) = -400<0$, which shows revenue is maximum after 10 days.
The farmer has 90 tons of cotton after 10 days and will get Rs.1,62,000 at the rate Rs.18,000 per ton.

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