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FUZZY PRE SEMI - λ CLOSED SETS IN FUZZY SCALABLE STRUCTURE SPACE

Dr.D.Amsaveni[1], S. Meenakshi[2] and J. Tamilmani[3]
[1]*Department of Mathematics (Assistant Professor), Sri Sarada College for Women, Salem - 636 016, Tamilnadu, India*
[2,3]*Department of Mathematics (Research Scholars),Sri Sarada College for Women, Salem - 636 016, Tamilnadu, India*

Abstract

In this paper, a new class of sets called FQ_X^{**} pre semi-λ closed sets in fuzzy scalable structure space is introduced and its possible interrelations with other type of FQ_X^{**} closed sets are studied with necessary counter examples. Similar types of interrelations for continuous functions are also been discussed.

Keywords: Fuzzy scalable structure space, FQ_X^{**} pre semi-λ closed set, FQ_X^{**} continuous function.
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I. Introduction

Fuzzy topological spaces are enriched by additional structures in order to give a more realistic representation of real life phenomena and computational process and at the same time, to provide for utilization of the powerful technique need a scale because all measurements and the majority of computations are performed in a definite space and problems of scalability are important both for physical theories and computational algorithms. The fuzzy concepts have penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [11]. The theory of fuzzy topological spaces was introduced and developed by Chang [6]. M.Burgin [4 , 5] introduced the concept of discontinuity structures in topological spaces and also investigated the concept of scale Q in a topological space. This structure serves as a scale for the initial topology on X. In this paper, the notion of FQ_X^{**} pre semi-λ closed sets and FQ_X^{**} continuous functions in fuzzy scalable structure spaces are introduced and studied. Some of their properties are investigated.

1. Preliminaries

Definition 1.1 [11]

Let X be a non-empty set and I be the unit interval. A fuzzy set in X is an element of the set I^X of all functions from X to I.

Definition 1.2 [6]

A fuzzy topology is a family T of fuzzy sets in (X, T) which satisfies the following conditions:

(a) $0, 1 \in T$;
(b) if $A, B \in T$, then $A \cap B \in T$;
(c) if $A_i \in T$ for each $i \in I$, then $\bigcup A_i \in T$.

Then, T is called a fuzzy topology for X and the pair (X, T) is a fuzzy topological space (abbreviated as fts). Every member of T is called a T-open fuzzy set. A fuzzy set is T-closed if and only if its complement is T-open.

Definition 1.3

A fuzzy set μ of a fts (X, T) is called:

1. a fuzzy pre-open set [3] if $\mu \leq \text{int}(cl(\mu))$ and a fuzzy pre-closed set if $cl(\text{int}(\mu)) \leq \mu$.
2. a fuzzy semi-open set [2] if $\mu \leq cl(\text{int}(\mu))$ and a fuzzy semi-closed set if $\text{int}(cl(\mu)) \leq \mu$.
3. a fuzzy semi-pre open set [9] if $\mu \leq cl(\text{int}(cl(\mu)))$ and a fuzzy semi-pre closed if $\text{int}(cl(\text{int}\mu))) \leq \mu$.
4. a fuzzy regular open set [2] if $\mu = \text{int}(cl(\mu))$ and a fuzzy regular closed set if $\mu = cl(\text{int}(\mu))$.
5. a fuzzy generalized open (fg-open) set [1] if $\text{int}(\mu) \leq \omega$ whenever $\mu \leq \omega$ and ω is fuzzy open set in (X,T). The complement of fuzzy g-open set is called fuzzy g-closed set in (X,T).
6. a fuzzy b-open (fb-open) set [2] iff $\mu \leq (\text{int}\,cl\mu) \vee (cl\,\text{int}\,\mu)$ and a fuzzy b-closed
7. (fb-closed) set iff $\mu \geq (cl\,\text{int}\,\mu) \wedge (\text{int}\,cl\mu)$.
8. a fuzzy t-set [10] if $\text{int}(\mu) = \text{int}\,cl(\mu)$.
9. a fuzzy B-set [10] if $\mu = \mu_1 \wedge \mu_2$ where μ_1 is fuzzy open and μ_2 is a fuzzy t-set.

a. The class of all fuzzy pre-open (resp. fuzzy semi-open, fuzzy semi-pre open, fuzzy regular open, fuzzy generalized open, fuzzy b-open, fuzzy t-set and fuzzy B set) sets in a fuzzy topological space (X, T) is denoted by FPO(X) (resp. FSO(X), FSPO(X), FRO(X), FGO(X), FbO(X), Ft(X) and FB(X)).

Definition 1.4 [7]

A subset B of a space X is a Λ-set if $B = B^{\Lambda}$, where $B^{\Lambda} = \wedge\{U, U > B, U \in T\}$.

Definition 1.5 [7]

A subset A of a space X is called λ-closed if A = B $\wedge$ C, where B is a Λ-set and C is a closed set.

Definition 1.6 [7]

A subset A of a space X is called λ-open if $A^c = X \backslash A$ is λ-closed.

Definition 1.7[9]

Let μ be a fuzzy set in a fts X. Then fuzzy semi pre - interior of μ is denoted and defined by spint(μ) $= \vee\{\omega : \omega \leq \mu, \mu \in FSPO(X)\}$.

Definition 1.8[8]

A fuzzy set μ of a fts X is called a fuzzy pre semi open set if spint(μ) $\geq \omega$, whenever $\mu \geq \omega$ and ω is a fuzzy g-closed set in (X,T). The class of all fuzzy pre semi open sets is denoted by FPSO(X).

The complement of a fuzzy pre semi open set is called a fuzzy pre semi closed set in (X, T).

Definition 1.9 [5]

Let X be a topological space with a topology Tx and TQ be a subset of Tx, i.e., TQ consists of open sets from X.

A scale or discontinuity structure Qx = Q on X is a mapping X $\rightarrow 2^{TQ}$ that satisfies the following conditions:

(SC1) For all points x from X, if A is an element of Q(x), then x$\in$ A.

(SC2) Any set from TQ belongs to some set from Q(x).

2. Fuzzy Pre semi - λ closed sets in fuzzy scalable structure space

Definition 2.1

Let X be a non empty set, $\tau_i : i \in J$ be the fuzzy topologies associated with X and let τ_X^* be the collection of all fuzzy open sets of (X, τ_i) and a function $Q_X : X \to \tau_X^*$ be the fuzzy scale.

A fuzzy scalable structure is denoted by Q_X^{**} and defined by $Q_X^{**} = \{0,1\} \cup Q_X^*$ where $Q_X^* = \{Q_X(x) : x \in X\}$.

Definition 2.2

Let X be a non empty set, τ_X^* be the collection of all fuzzy open sets of X and Q_X^{**} be a fuzzy scalable structure on X. Then, the triad (X, τ_X^*, Q_X^{**}) is called a fuzzy scalable structure space.

Every member of Q_X^{**} is called a fuzzy Q_X^{**} open set (briefly, FQ_X^{**} open). Its complement is said to be a fuzzy Q_X^{**} closed set (briefly, FQ_X^{**} closed).

Definition 2.3

Let (X, τ_X^*, Q_X^{**}) be a fuzzy scalable structure space. The fuzzy Q_X^{**} interior and fuzzy Q_X^{**} closure of a fuzzy set μ of X are defined and denoted as follows:

(i) $FQ_X^{**}\operatorname{int}(\mu) = \vee\{\omega : \omega\, is\, a\, FQ_X^{**}\, open\ set\ and\ \omega \le \mu\}$.

(ii) $FQ_X^{**}\, cl(\mu) = \wedge\{\omega : \omega\, is\, a\, FQ_X^{**}\, closed\ set\ and\ \omega \ge \mu\}$.

Definition 2.4

A fuzzy subset μ of a fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is called:

1. a fuzzy Q_X^{**} pre-open (in short, FQ_X^{**} pre-open) set, if $\mu \le FQ_X^{**}\operatorname{int}(FQ_X^{**}cl(\mu))$ and a fuzzy Q_X^{**} pre-closed (in short, FQ_X^{**} pre-closed) set if $FQ_X^{**}cl(FQ_X^{**}\operatorname{int}(\mu)) \le \mu$.
2. a fuzzy Q_X^{**} semi-open (in short, FQ_X^{**} semi-open) set if $\mu \le FQ_X^{**}cl(FQ_X^{**}\operatorname{int}(\mu))$ and a fuzzy Q_X^{**} semi-closed (in short, FQ_X^{**} semi-closed) set if $FQ_X^{**}\operatorname{int}(FQ_X^{**}cl(\mu)) \le \mu$.
3. a fuzzy Q_X^{**} semi-pre open (in short, FQ_X^{**} sp-open) set if $\mu \le FQ_X^{**}cl(FQ_X^{**}\operatorname{int}(FQ_X^{**}cl(\mu)))$ and a fuzzy Q_X^{**} semi-pre closed (in short, FQ_X^{**} sp-closed) if $FQ_X^{**}\operatorname{int}(FQ_X^{**}cl(FQ_X^{**}\operatorname{int}\mu))) \le \mu$.
4. a fuzzy Q_X^{**} regular open (in short, FQ_X^{**} regular-open) set if $\mu = FQ_X^{**}\operatorname{int}(FQ_X^{**}cl(\mu))$ and a fuzzy Q_X^{**} regular closed (in short, FQ_X^{**} regular-closed) set if $\mu = FQ_X^{**}cl(FQ_X^{**}\operatorname{int}(\mu))$.
5. a fuzzy Q_X^{**} generalized open (in short, FQ_X^{**} g-open) set if $FQ_X^{**}\operatorname{int}(\mu) \le \omega$ whenever $\mu \le \omega$ and ω is fuzzy Q_X^{**} open set in (X, Q_X^{**}). The complement of fuzzy Q_X^{**} g-open set is called fuzzy Q_X^{**} g-closed (in short, FQ_X^{**} g-closed) set in (X, τ_X^*, Q_X^{**}).
6. a fuzzy Q_X^{**} b-open (in short, FQ_X^{**} b-open) set if $\mu \le (FQ_X^{**}\operatorname{int} FQ_X^{**}cl\mu) \vee (FQ_X^{**}clFQ_X^{**}\operatorname{int}\mu)$ and a fuzzy Q_X^{**} b-closed (in short FQ_X^{**} b-closed) set if $\mu \ge (FQ_X^{**}clFQ_X^{**}\operatorname{int}\mu) \wedge (FQ_X^{**}\operatorname{int} FQ_X^{**}cl\mu)$.
7. a fuzzy Q_X^{**} t-set (in short, FQ_X^{**} t-set) if $FQ_X^{**}\operatorname{int}(\mu) = FQ_X^{**}\operatorname{int} FQ_X^{**}cl(\mu)$.
8. a fuzzy Q_X^{**} B-set (in short, FQ_X^{**} B-set) if $\mu = \mu_1 \wedge \mu_2$ where μ_1 is a FQ_X^{**} t - set and μ_2 is a FQ_X^{**} open set.

The class of all FQ_X^{**} pre-open (resp. FQ_X^{**} semi-open, FQ_X^{**} semi-pre open, FQ_X^{**} regular open, FQ_X^{**} generalized open, FQ_X^{**} b open, FQ_X^{**} t-set and FQ_X^{**} B set) sets in a fuzz scalable structure space (X, τ_X^*, Q_X^{**}) is denoted by FQ_X^{**} PO(X) (respectively FQ_X^{**} SO(X), FQ_X^{**} SPO(X), FQ_X^{**} RO(X), FQ_X^{**} GO(X), FQ_X^{**} bO(X), FQ_X^{**} t(X) and FQ_X^{**} B(X)).

Definition 2.5

A fuzzy subset μ of a fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is called:

1. a fuzzy Q_X^{**} - Λ set (in short, FQ_X^{**} - Λ set) if $\mu = FQ_X^{**}\mu^{\Lambda}$, where $FQ_X^{**}\mu^{\Lambda} = \wedge\{\omega, \omega > \mu, \omega \in Q_X^{**}\}$
2. a fuzzy Q_X^{**} pre semi - Λ set (in short, FQ_X^{**} pre semi - Λ set) if $\mu = FQ_X^{**} PS\, \mu^{\Lambda}$, where $FQ_X^{**} PS\, \mu^{\Lambda} = \wedge\{\omega, \omega > \mu, \omega \in FQ_X^{**} PSO(X)\}$
3. a fuzzy Q_X^{**} regular - Λ set (in short, FQ_X^{**} regular - Λ set) if $\mu = FQ_X^{**} R\, \mu^{\Lambda}$, where $FQ_X^{**} R\, \mu^{\Lambda} = \wedge\{\omega, \omega > \mu, \omega \in FQ_X^{**} RO(X)\}$
4. a fuzzy Q_X^{**} b - Λ set (in short, FQ_X^{**} b - Λ set) if $\mu = FQ_X^{**} b\, \mu^{\Lambda}$, where $FQ_X^{**} b\, \mu^{\Lambda} = \wedge\{\omega, \omega > \mu, \omega \in FQ_X^{**} bO(X)\}$

Definition 2.6

A fuzzy subset μ of a fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is called:

1. a fuzzy Q_X^{**} λ-closed (in short, FQ_X^{**} λ- closed) if $\mu = \mu_1 \wedge \mu_2$, where μ_1 is a FQ_X^{**} Λ-set and μ_2 is a FQ_X^{**} closed set.
2. a fuzzy Q_X^{**} pre semi λ-closed (in short, FQ_X^{**} pre semi λ-closed) if $\mu = \mu_1 \wedge \mu_2$, where μ_1 is a FQ_X^{**} pre semi Λ-set and μ_2 is a FQ_X^{**} closed set.
3. a fuzzy Q_X^{**} regular λ-closed (in short, FQ_X^{**} regular λ-closed) if $\mu = \mu_1 \wedge \mu_2$, where μ_1 is a FQ_X^{**} regular Λ-set and μ_2 is a FQ_X^{**} closed set.
4. a fuzzy Q_X^{**} b - λ closed (in short, FQ_X^{**} b - λ closed) if $\mu = \mu_1 \wedge \mu_2$, where μ_1 is a FQ_X^{**} b - Λ set and μ_2 is a FQ_X^{**} closed set.

The complement of a FQ_X^{**} λ-closed (resp. FQ_X^{**} pre semi λ-closed, FQ_X^{**} regular λ-closed and FQ_X^{**} b - λ closed) set is called a FQ_X^{**} λ-open(resp. FQ_X^{**} pre semi λ- open , FQ_X^{**} regular λ-open and FQ_X^{**} b - λ open).

Proposition 2.1

Every FQ_X^{**}-closed set in fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is a FQ_X^{**} semi pre closed set.

Remark 2.1

The converse part of the above Proposition 2.1 need not be true as shown in the following example.

Example 2.1

Let $X = \{a, b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1 = \{0, 1, \mu_1, \mu_2\}$ and $\tau_2 = \{0, 1, \mu_3\}$ be the fuzzy topologies on X where $\mu_1, \mu_2, \mu_3 \in I^X$ defined as $\mu_1(a) = 0.7, \mu_1(b) = 0.6$; $\mu_2(a) = 0.3, \mu_2(b) = 0.2$; $\mu_3(a) = 0.5, \mu_3(b) = 0.4$. Now, define $Q_X : X \to \tau_X^*$ by $Q_X(a) = \mu_1$; $Q_X(b) = \mu_3$. Then, Q_X is a fuzzy scale and $Q_X^* = \{\mu_1, \mu_3\}$. Let $Q_X^{**} = \{0, 1, \mu_1, \mu_3\}$ be a fuzzy scalable structure on X. Clearly, the triad (X, τ_X^*, Q_X^{**}) is fuzzy scalable structure space. Here the fuzzy set μ_2 is a FQ_X^{**} semi pre closed but not FQ_X^{**} closed.

Proposition 2.2

Every FQ_X^{**} semi pre closed set in fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is a FQ_X^{**} pre semi closed set.

Remark 2.2

The converse part of the above Proposition 2.2 need not be true as shown in the following example.

Example 2.2

Let $X=\{a,b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1=\{0,1,\mu_1,\mu_2\}$ and $\tau_2=\{0,1,\mu_3\}$ be the fuzzy topologies on X where $\mu_1,\mu_2,\mu_3\in I^X$ defined as $\mu_1(a)=0.2,\mu_1(b)=0.3$; $\mu_2(a)=0.7,\mu_2(b)=0.7$; $\mu_3(a)=0.3,\mu_3(b)=0.4$. Now, define $Q_X : X\to\tau_X^*$ by $Q_X(a)=\mu_1$; $Q_X(b)=\mu_3$. Then, Q_X is a fuzzy scale and $Q_X^*=\{\mu_1,\mu_3\}$. Let $Q_X^{**}=\{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X. Clearly, the triad (X,τ_X^*,Q_X^{**}) is fuzzy scalable structure space. Here the fuzzy set μ_2 is a FQ_X^{**} pre semi closed but not semi pre FQ_X^{**} closed.

Proposition 2.3

Every FQ_X^{**} closed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}) is a FQ_X^{**} pre semi closed set.

Remark 2.3

The converse part of the above Proposition 2.3 need not be true as shown in the following example.

Example 2.3

In example 2.1, the fuzzy set μ_2 is a FQ_X^{**} pre semi closed but not FQ_X^{**} closed.

Proposition 2.4

Every FQ_X^{**} regular closed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}) is a FQ_X^{**} semi pre closed set.

Remark 2.4

The converse part of the above Proposition 2.4 need not be true as shown in the following example.

Example 2.4

In example 2.1, the fuzzy set μ_2 is a FQ_X^{**} semi pre closed but not FQ_X^{**} regular closed.

Proposition 2.5

Every FQ_X^{**} regular closed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}) is a FQ_X^{**} pre semi closed set.

Remark 2.5

The converse part of the above Proposition 2.5 need not be true as shown in the following example.

Example 2.5

In example 2.1, the fuzzy set μ_2 is FQ_X^{**} pre semi closed but not FQ_X^{**} regular closed.

Proposition 2.6

Every FQ_X^{**} b - closed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}) is a FQ_X^{**} semi pre closed set.

Remark 2.6

The converse part of the above Proposition 2.6 need not be true as shown in the following example.

Example 2.6

In example 2.1, the fuzzy set μ_2 is a FQ_X^{**} semi pre closed but not FQ_X^{**} b-closed.

Proposition 2.7

Every FQ_X^{**} b - closed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}) is a FQ_X^{**} pre semi closed set.

Remark 2.7

The converse part of the above Proposition 2.7 need not be true as shown in the following example.

Example 2.7

In example 2.1, the fuzzy set μ_2 is a FQ_X^{**} pre semi closed but not FQ_X^{**} b-closed.

Proposition 2.8

Every FQ_X^{**} closed set in fuzzy scalable structure space (X, τ_X^*, Q_X^{**}) is:

1. a FQ_X^{**} pre semi-λ closed set
2. a FQ_X^{**} t-set
3. a FQ_X^{**} B-set
4. a FQ_X^{**} b-λ closed set
5. a FQ_X^{**} λ- closed set
6. a FQ_X^{**} regular- λ closed set

Remark 2.8

The converse part of the above Proposition 2.8 need not be true as shown in the following examples.

Example 2.8

Let $X=\{a,b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1=\{0,1,\mu_1,\mu_2,\mu_4,\mu_5\}$ and $\tau_2=\{0,1,\mu_3\}$ be the fuzzy topologies on X where $\mu_1,\mu_2,\mu_3,\mu_4\in I^X$ defined as $\mu_1(a)=0.4, \mu_1(b)=0.3$; $\mu_2(a)=0.4, \mu_2(b)=0.1$; $\mu_3(a)=0.8$, $\mu_3(b)=0.1$; $\mu_4(a)=0.3$, $\mu_4(b)=0.3$; $\mu_5(a)=0.3, \mu_5(b)=0.1$. Now, define $Q_X: X\to\tau_X^*$ by $Q_X(a)=\mu_1$; $Q_X(b)=\mu_3$. Then, Q_X is a fuzzy scale and $Q_X^*=\{\mu_1,\mu_3\}$. Let $Q_X^{**}=\{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X. Clearly, the triad (X,τ_X^*,Q_X^{**}) is fuzzy scalable structure space. Here the fuzzy set ω defined by $\omega(a)=0.4, \omega(b)=0.4$ is a FQ_X^{**} pre semi-λ closed, FQ_X^{**} t-set, FQ_X^{**} B –set and FQ_X^{**} b -λ set but not FQ_X^{**} closed set. The fuzzy set μ_2 is FQ_X^{**} -λ closed but not FQ_X^{**} closed.

Example 2.9

Let $X=\{a,b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1=\{0,1,\mu_1,\mu_2,\mu_4\}$ and $\tau_2=\{0,1,\mu_3\}$ be the fuzzy topologies on X where $\mu_1,\mu_2,\mu_3,\mu_4\in I^X$ defined as $\mu_1(a)=0.6, \mu_1(b)=0.4$; $\mu_2(a)=0.1, \mu_2(b)=0.3$; $\mu_3(a)=0.9$, $\mu_3(b)=0.4; and\ \mu_4(a)=0.3$, $\mu_4(b)=0.4$. Now, define $Q_X: X\to\tau_X^*$ by $Q_X(a)=\mu_2$; $Q_X(b)=\mu_3$. Then, Q_X is a fuzzy scale and $Q_X^*=\{\mu_2,\mu_3\}$. Let $Q_X^{**}=\{0,1,\mu_2,\mu_3\}$ be a fuzzy scalable structure on X. Clearly, the triad (X,τ_X^*,Q_X^{**}) is fuzzy scalable structure space. Here the fuzzy set ω defined by $\omega(a)=0.1, \omega(b)=0.4$ is a FQ_X^{**} regular-λ closed but not FQ_X^{**} closed.

Proposition 2.9

Every FQ_X^{**} λ- closed set is a FQ_X^{**} pre semi- λclosed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}).

Remark 2.9

The converse part of the above Proposition 2.9 need not be true as shown in the following example.

Example 2.10

In example 2.8 the fuzzy set μ_4 is a FQ_X^{**} pre semi-λ closed but not FQ_X^{**} λ- closed set in (X,τ_X^*,Q_X^{**}).

Proposition 2.10

Every FQ_X^{**} regular - λclosed set is a FQ_X^{**} pre semi- λclosed set in fuzzy scalable structure space (X,τ_X^*,Q_X^{**}).

Remark 2.10

The converse part of the above Proposition 2.10 need not be true as shown in the example.

Example 2.11

In example 2.8 the fuzzy set μ_1 is a FQ_X^{**} pre semi-λ closed but not FQ_X^{**} regular - λclosed set in (X, τ_X^*, Q_X^{**}).

Proposition 2.11

Every FQ_X^{**} b -λclosed set is a FQ_X^{**} pre semi- λclosed set in fuzzy scalable structure space (X, τ_X^*, Q_X^{**}).

Remark 2.11

The converse part of the above Proposition 2.11 need not be true as shown in the following example.

Example 2.12

In example 2.8 the fuzzy set ω defined by $\omega(a)=0, \omega(b)=0.6$ is a FQ_X^{**} pre semi-λ closed but not FQ_X^{**} b - λclosed set in (X, τ_X^*, Q_X^{**}).

Proposition 2.12

Every FQ_X^{**} t-set is a FQ_X^{**} B-set in the fuzzy scalable structure space (X, τ_X^*, Q_X^{**}).

Remark 2.12

The converse part of the above Proposition 2.12 need not be true as shown in the following example.

Example 2.13

Let $X=\{a,b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1=\{0,1,\mu_1,\mu_2\}$ and $\tau_2=\{0,1,\mu_2,\mu_3\}$ be the fuzzy topologies on X where $\mu_1,\mu_2,\mu_3 \in I^X$ defined as $\mu_1(a)=0.2, \mu_1(b)=0.3$; $\mu_2(a)=0.5, \mu_2(b)=0.6$ *and* $\mu_3(a)=0.1$, $\mu_3(b)=0.2$. Now, define $Q_X : X \to \tau_X^*$ by $Q_X(a)=\mu_2$; $Q_X(b)=\mu_2$. Then, Q_X is a fuzzy scale and $Q_X^* = \{\mu_2\}$. Let $Q_X^{**}=\{0,1,\mu_2\}$ be a fuzzy scalable structure on X. Clearly, the triad (X, τ_X^*, Q_X^{**}) is a fuzzy scalable structure space. Here the fuzzy set μ_2 is a FQ_X^{**} B-set but not a FQ_X^{**} t - set.

Proposition 2.13

Every FQ_X^{**} regular - λ closed set is a FQ_X^{**} t -set in the fuzzy scalable structure space (X, τ_X^*, Q_X^{**}).

Remark 2.13

The converse part of the above Proposition 2.13 need not be true as shown in the following example.

Example 2.14

Let $X=\{a,b\}$ be a non empty set and τ_X^* be the collection of all fuzzy open sets in X. Let $\tau_1=\{0,1,\mu_1,\mu_2\}$ and $\tau_2=\{0,1,\mu_3\}$ be the fuzzy topologies on X where $\mu_1,\mu_2,\mu_3 \in I^{X_1}$ are defined as $\mu_1(a)=0.6, \mu_1(b)=0.2$; $\mu_2(a)=0.7, \mu_2(b)=0.2$; $\mu_3(a)=0.1$, $\mu_3(b)=0.8$. Now define $Q_X : X \to \tau_X^*$ by $Q_{X_1}(a)=\mu_2$; $Q_{X_1}(b)=\mu_3$. Then, Q_X is a fuzzy scale and $Q_X^*=\{\mu_2,\mu_3\}$. Let $Q_X^{**}=\{0,1,\mu_2,\mu_3\}$ be a fuzzy scalable structure on X. Clearly, the triad (X, τ_X^*, Q_X^{**}) is a fuzzy scalable structure space. Here the fuzzy set ω defined by $\omega(a)=0.8, \omega(b)=0.2$ is a FQ_X^{**} t -set but not FQ_X^{**} regular closed set.

Proposition 2.14

Every FQ_X^{**} regular - λ closed set is a FQ_X^{**} b - λ closed set in the fuzzy scalable structure space (X, τ_X^*, Q_X^{**}).

Remark 2.14

The converse part of the above Proposition 2.14 need not be true as shown in the following example.

Example 2.15

In example 2.14, the fuzzy set ω defined by $\omega(a) = 0.3, \omega(b) = 0.2$ is a FQ_X^{**} b - λ closed set but not a FQ_X^{**} regular - λ closed set.

Proposition 2.15

Every FQ_X^{**} λ - closed set is a FQ_X^{**} b - λ closed set in the fuzzy scalable structure space (X, τ_X^*, Q_X^{**}).

Remark 2.15

The converse part of the above Proposition 2.15 need not be true as shown in the following example.

Example 2.16

In example 2.14, the fuzzy set ω defined by $\omega(a) = 0.8, \omega(b) = 0.2$ is a FQ_X^{**} b - λ closed set but not a FQ_X^{**} regular - λ closed set.

Remark 2.16

From the above discussion we have the following implications are true:

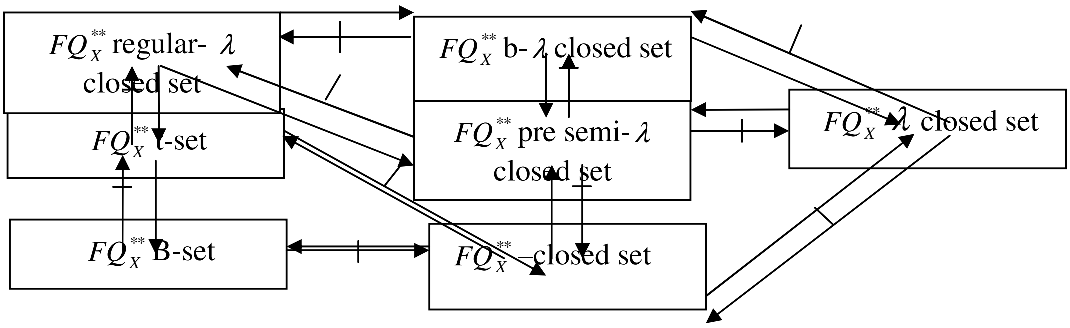

Figure: 1

Relationship between FQ_X^{**} - pre semi λ closed set and other existing FQ_X^{**} - closed sets

3. FQ_X^{**} Pre semi - λ continuity in fuzzy scalable structure spaces

Definition 3.1

Let $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$ and $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ be any two fuzzy scalable structure spaces. Then the function f : $(X_1, \tau_{X_1}^*, Q_{X_1}^{**}) \rightarrow (X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ is said to be FQ_X^{**} continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ - closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.

Definition 3.2

Let $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$ and $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ be any two fuzzy scalable structure spaces. Then the function f : $(X_1, \tau_{X_1}^*, Q_{X_1}^{**}) \rightarrow (X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ is said to be:

1. FQ_X^{**} pre semi- λ continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ pre semi- λ - closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.

2. FQ_X^{**} t -continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ t -closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.
3. FQ_X^{**} B -continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ B -closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.
4. FQ_X^{**} b - λ continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ b - λ closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.
5. FQ_X^{**} λ - continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a $FQ_{X_1}^{**}$ λ - closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.
6. FQ_X^{**} regular - λ continuous if for every $FQ_{X_2}^{**}$ - closed set in $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ there exists a fuzzy $FQ_{X_1}^{**}$ regular - λ closed set in $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$.

Proposition 3.1

Let $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$ and $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau_{X_1}^*, Q_{X_1}^{**}) \to (X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ is FQ_X^{**} continuous function then it is:

1. a FQ_X^{**} pre semi- λ continuous function.
2. a FQ_X^{**} t-continuous function.
3. a FQ_X^{**} B-continuous function.
4. a FQ_X^{**} b- λ continuous function.
5. a FQ_X^{**} λ - continuous function.
6. a FQ_X^{**} regular- λ continuous function.

Remark 3.1

The converse part of the above Proposition 3.1 need not be true as shown in the following examples.

Example 3.1

Let $X_1 = \{a, b\}$ be a non empty set and $\tau_{X_1}^*$ be the collection of all fuzzy open sets in X₁. Let $\tau_1 = \{0,1,\mu_1,\mu_2,\mu_4,\mu_5\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3,\mu_4,\mu_5 \in I^{X_1}$ are defined as $\mu_1(a) = 0.4, \mu_1(b) = 0.3$; $\mu_2(a) = 0.4, \mu_2(b) = 0.1$; $\mu_3(a) = 0.8$, $\mu_3(b) = 0.1; \mu_4(a) = 0.3$, $\mu_4(b) = 0.3$; $\mu_5(a) = 0.3, \mu_5(b) = 0.1$. Now, define $Q_{X_1} : X_1 \to \tau_{X_1}^*$ by $Q_{X_1}(a) = \mu_1$; $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q_{X_1}^* = \{\mu_1, \mu_3\}$. Let $Q_{X_1}^{**} = \{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau_{X_1}^*, Q_{X_1}^{**})$ is a fuzzy scalable structure space.

Let $X_2 = \{u, v\}$ be a non empty set and $\tau_{X_2}^*$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ are defined as $\omega_1(u) = 0.2, \omega_1(v) = 0.2$; $\omega_2(u) = 0.4, \omega_2(v) = 0.3$; $\omega_3(u) = 0.6$, $\omega_3(v) = 0.7$. Now, define $Q_{X_2} : X_2 \to \tau_{X_2}^*$ by $Q_{X_2}(u) = \omega_1$; $Q_{X_2}(v) = \omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q_{X_2}^* = \{\omega_1, \omega_3\}$. Let $Q_{X_2}^{**} = \{0,1,\omega_1,\omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau_{X_1}^*, Q_{X_1}^{**}) \to (X_2, \tau_{X_2}^*, Q_{X_2}^{**})$ be a FQ_X^{**} continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ_X^{**} pre semi- λ continuous and FQ_X^{**} b - λ continuous but not FQ_X^{**} continuous.

Example 3.2

Let $X_1 = \{a,b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0,1,\mu_1,\mu_2\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3 \in I^{X_1}$ are defined as $\mu_1(a) = 0.6, \mu_1(b) = 0.2$; $\mu_2(a) = 0.7, \mu_2(b) = 0.2$; $\mu_3(a) = 0.1$, $\mu_3(b) = 0.8$. Now define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a) = \mu_2$; $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_2, \mu_3\}$. Let $Q^{**}_{X_1} = \{0,1,\mu_2,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u,v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ are defined as $\omega_1(u) = 0.3, \omega_1(v) = 0.8$; $\omega_2(u) = 0.2, \omega_2(v) = 0.3$; $\omega_3(u) = 0.6$, $\omega_3(v) = 0.7$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u) = \omega_1$; $Q_{X_2}(v) = \omega_1$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1\}$. Let $Q^{**}_{X_2} = \{0,1,\omega_1\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a FQ^{**}_X continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X regular - λ continuous but not FQ^{**}_X continuous.

Example 3.3

Let $X_1 = \{a,b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0,1,\mu_1,\mu_2\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3 \in I^{X_1}$ defined as $\mu_1(a) = 0.5, \mu_1(b) = 0.6$; $\mu_2(a) = 0.2, \mu_2(b) = 0.1$; $\mu_3(a) = 0.3$, $\mu_3(b) = 0.2$. Now, define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a) = \mu_1$; $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_1, \mu_3\}$. Let $Q^{**}_{X_1} = \{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u,v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ defined as $\omega_1(u) = 0.4, \omega_1(v) = 0.3$; $\omega_2(u) = 0.3, \omega_2(v) = 0.2$; $\omega_3(u) = 0.7$, $\omega_3(v) = 0.6$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u) = \omega_1$; $Q_{X_2}(v) = \omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1, \omega_3\}$. Let $Q^{**}_{X_2} = \{0,1,\omega_1,\omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a FQ^{**}_X continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X t-continuous and FQ^{**}_X B-continuous but not FQ^{**}_X continuous.

Example 3.4

Let $X_1 = \{a,b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0,1,\mu_1,\mu_2,\mu_4,\mu_5\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3,\mu_4,\mu_5 \in I^{X_1}$ defined as $\mu_1(a) = 0.4, \mu_1(b) = 0.3$; $\mu_2(a) = 0.4, \mu_2(b) = 0.1$; $\mu_3(a) = 0.8$, $\mu_3(b) = 0.1$; $\mu_4(a) = 0.3$, $\mu_4(b) = 0.3$; $\mu_5(a) = 0.3, \mu_5(b) = 0.1$. Now, define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a) = \mu_1$; $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_1, \mu_3\}$. Let $Q^{**}_{X_1} = \{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u, v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ defined as $\omega_1(u) = 0.6, \omega_1(v) = 0.9$; $\omega_2(u) = 0.4, \omega_2(v) = 0.3$; $\omega_3(u) = 0.8$, $\omega_3(v) = 0.9$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u) = \omega_1$; $Q_{X_2}(v) = \omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1, \omega_3\}$. Let $Q^{**}_{X_2} = \{0,1,\omega_1,\omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a FQ^{**}_X continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X λ - continuous but not FQ^{**}_X continuous.

Proposition 3.2

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X λ - continuous then it is FQ^{**}_X pre semi- λ continuous.

Remark 3.2

The converse part of the above Proposition 3.2 need not be true as shown in the following example.

Example 3.5

In example 3.1, the function f is FQ^{**}_X pre semi λ - continuous but not FQ^{**}_X λ - continuous.

Proposition 3.3

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X regular λ - continuous then it is FQ^{**}_X pre semi- λ continuous.

Remark 3.3

The converse part of the above Proposition 3.3 need not be true as shown in the following example.

Example 3.6

In example 3.1, the function f is FQ^{**}_X pre semi λ - continuous but not FQ^{**}_X regular - λ continuous.

Proposition 3.4

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X b - λ continuous then it is FQ^{**}_X pre semi- λ continuous.

Remark 3.4

The converse part of the above Proposition 3.4 need not be true as shown in the following example.

Example 3.7

Let $X_1 = \{a, b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0,1,\mu_1,\mu_2,\mu_4,\mu_5\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3,\mu_4,\mu_5 \in I^{X_1}$ are defined as $\mu_1(a) = 0.4, \mu_1(b) = 0.3$; $\mu_2(a) = 0.4, \mu_2(b) = 0.1$; $\mu_3(a) = 0.8$, $\mu_3(b) = 0.1$; $\mu_4(a) = 0.3$, $\mu_4(b) = 0.3$; $\mu_5(a) = 0.3, \mu_5(b) = 0.1$. Now, define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a) = \mu_1$; $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_1, \mu_3\}$. Let $Q^{**}_{X_1} = \{0,1,\mu_1,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u, v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ defined as $\omega_1(u)=1, \omega_1(v)=0.4$; $\omega_2(u)=0.4, \omega_2(v)=0.3$; $\omega_3(u)=0.3$, $\omega_3(v)=0.3$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u)=\omega_1$; $Q_{X_2}(v)=\omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1,\omega_3\}$. Let $Q^{**}_{X_2} = \{0,1,\omega_1,\omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a fuzzy FQ^{**}_X function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X pre semi-λ continuous but not FQ^{**}_X b-λ continuous.

Proposition 3.5

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X t- continuous then it is FQ^{**}_X B -continuous.

Remark 3.5

The converse part of the above Proposition 3.5 need not be true as shown in the following example.

Example 3.8

In example 2.13, define the mapping f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ by f(a) = a and f(b) = b. Then f is FQ^{**}_X B - continuous but not FQ^{**}_X t -continuous.

Proposition 3.6

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X regular - λ continuous then it is FQ^{**}_X t - continuous.

Remark 3.6

The converse part of the above Proposition 3.6 need not be true as shown in the following example.

Example 3.9

Let $X_1 = \{a, b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0,1,\mu_1,\mu_2\}$ and $\tau_2 = \{0,1,\mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1,\mu_2,\mu_3 \in I^{X_1}$ are defined as $\mu_1(a)=0.6, \mu_1(b)=0.2$; $\mu_2(a)=0.7, \mu_2(b)=0.2$; $\mu_3(a)=0.1$, $\mu_3(b)=0.8$. Now define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a)=\mu_2$; $Q_{X_1}(b)=\mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_2,\mu_3\}$. Let $Q^{**}_{X_1} = \{0,1,\mu_2,\mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u, v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0,1,\omega_1,\omega_2\}$ and $\tau_4 = \{0,1,\omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ are defined as $\omega_1(u)=0.2, \omega_1(v)=0.8$; $\omega_2(u)=0.2, \omega_2(v)=0.3$; $\omega_3(u)=0.9$, $\omega_3(v)=0.8$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u)=\omega_1$; $Q_{X_2}(v)=\omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1,\omega_3\}$. Let $Q^{**}_{X_2} = \{0,1,\omega_1,\omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let f : $(X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a FQ^{**}_X continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X t - continuous but not FQ^{**}_X regular - λ continuous.

Proposition 3.7

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function $f: (X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X regular - λ continuous then it is FQ^{**}_X b - λ continuous.

Remark 3.7

The converse part of the above Proposition 3.7 need not be true as shown in the following example.

Example 3.10

Let $X_1 = \{a, b\}$ be a non empty set and $\tau^*_{X_1}$ be the collection of all fuzzy open sets in X_1. Let $\tau_1 = \{0, 1, \mu_1, \mu_2\}$ and $\tau_2 = \{0, 1, \mu_3\}$ be the fuzzy topologies on X_1 where $\mu_1, \mu_2, \mu_3 \in I^{X_1}$ are defined as $\mu_1(a) = 0.6, \mu_1(b) = 0.2;$ $\mu_2(a) = 0.7, \mu_2(b) = 0.2;$ $\mu_3(a) = 0.1,$ $\mu_3(b) = 0.8$. Now define $Q_{X_1} : X_1 \to \tau^*_{X_1}$ by $Q_{X_1}(a) = \mu_2;$ $Q_{X_1}(b) = \mu_3$. Then, Q_{X_1} is a fuzzy scale and $Q^*_{X_1} = \{\mu_2, \mu_3\}$. Let $Q^{**}_{X_1} = \{0, 1, \mu_2, \mu_3\}$ be a fuzzy scalable structure on X_1. Clearly, the triad $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ is a fuzzy scalable structure space.

Let $X_2 = \{u, v\}$ be a non empty set and $\tau^*_{X_2}$ be the collection of all fuzzy open sets in X_2. Let $\tau_3 = \{0, 1, \omega_1, \omega_2\}$ and $\tau_4 = \{0, 1, \omega_3\}$ be the fuzzy topologies on X_2 where $\omega_1, \omega_2, \omega_3 \in I^{X_2}$ are defined as $\omega_1(u) = 0.3, \omega_1(v) = 0.8;$ $\omega_2(u) = 0.2, \omega_2(v) = 0.3;$ $\omega_3(u) = 0.9,$ $\omega_3(v) = 0.7$. Now, define $Q_{X_2} : X_2 \to \tau^*_{X_2}$ by $Q_{X_2}(u) = \omega_1;$ $Q_{X_2}(v) = \omega_3$. Then, Q_{X_2} is a fuzzy scale and $Q^*_{X_2} = \{\omega_1, \omega_3\}$. Let $Q^{**}_{X_2} = \{0, 1, \omega_1, \omega_3\}$ be a fuzzy scalable structure on X_2. Clearly, the triad $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is a fuzzy scalable structure space.

Let $f: (X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be a FQ^{**}_X continuous function defined by f(a) = u and f(b) = v. Then the function 'f' is FQ^{**}_X b - λ continuous but not FQ^{**}_X regular - λ continuous.

Proposition 3.8

Let $(X_1, \tau^*_{X_1}, Q^{**}_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ be any two fuzzy scalable structure spaces. If a function $f: (X_1, \tau^*_{X_1}, Q^{**}_{X_1}) \to (X_2, \tau^*_{X_2}, Q^{**}_{X_2})$ is FQ^{**}_X λ - continuous then it is FQ^{**}_X b - λ continuous.

Remark 3.8

The converse part of the above Proposition 3.8 need not be true as shown in the following example.

Example 3.11

In example 3.10, the function f is FQ^{**}_X b - λ continuous but not FQ^{**}_X λ continuous.

Remark 3.9

From the above discussion we have the following implications are true:

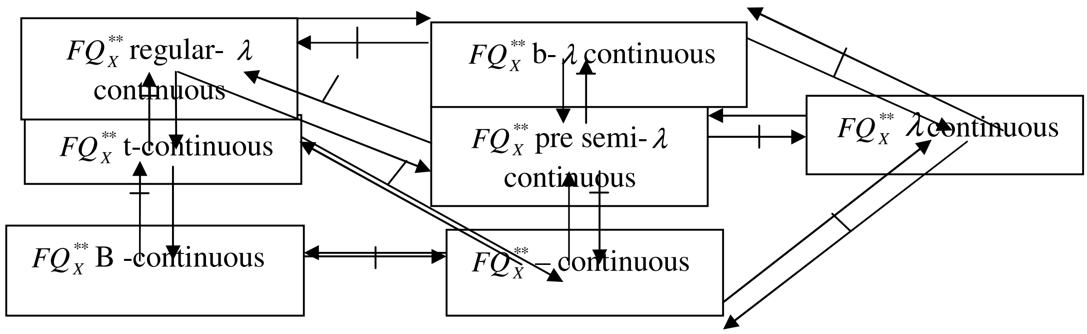

Figure: 2 Relationship between FQ^{**}_X - pre semi λ continuity and other existing FQ^{**}_X continuities

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