FUZZY PRE SEMI - $\lambda$ CLOSED SETS IN FUZZY SCALABLE STRUCTURE SPACE

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Abstract

In this paper, a new class of sets called $FQ_x^{\pre semi - \lambda}$ closed sets in fuzzy scalable structure space is introduced and its possible interrelations with other type of $FQ_x^\tau$ closed sets are studied with necessary counter examples. Similar types of interrelations for continuous functions are also been discussed.

Keywords: Fuzzy scalable structure space, $FQ_x^{\pre semi - \lambda}$ closed set, $FQ_x^\tau$ continuous function.

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I. Introduction

Fuzzy topological spaces are enriched by additional structures in order to give a more realistic representation of real life phenomena and computational process and at the same time, to provide for utilization of the powerful technique need a scale because all measurements and the majority of computations are performed in a definite space and problems of scalability are important both for physical theories and computational algorithms. The fuzzy concepts have penetrated almost all branches of Mathematics since the introduction of the concept of fuzzy set by Zadeh [11]. The theory of fuzzy topological spaces was introduced and developed by Chang [6]. M.Burgin [4, 5] introduced the concept of discontinuity structures in topological spaces and also investigated the concept of scale $Q$ in a topological space. This structure serves as a scale for the initial topology on $X$. In this paper, the notion of $FQ_x^{\pre semi - \lambda}$ closed sets and $FQ_x^\tau$ continuous functions in fuzzy scalable structure spaces are introduced and studied. Some of their properties are investigated.

1. Preliminaries

Definition 1.1 [11]

Let $X$ be a non-empty set and $I$ be the unit interval. A fuzzy set in $X$ is an element of the set $I^X$ of all functions from $X$ to $I$.

Definition 1.2 [6]

A fuzzy topology is a family $T$ of fuzzy sets in $(X, T)$ which satisfies the following conditions:

(a) $0, 1 \in T$;
(b) if $A, B \in T$, then $A \cap B \in T$;
(c) if $A_i \in T$ for each $i \in I$, then $\bigcup A_i \in T$. 
Then, $T$ is called a fuzzy topology for $X$ and the pair $(X, T)$ is a fuzzy topological space (abbreviated as fts). Every member of $T$ is called a $T$-open fuzzy set. A fuzzy set is $T$-closed if and only if its complement is $T$-open.

**Definition 1.3**

A fuzzy set $\mu$ of a fts $(X, T)$ is called:

1. a fuzzy pre-open set [3] if $\mu \leq \text{int}(cl(\mu))$ and a fuzzy pre-closed set if $\mu = cl(\text{int}(\mu))$. 
2. a fuzzy semi-open set [2] if $\mu \leq cl(\text{int}(\mu))$ and a fuzzy semi-closed set if $\text{int}(cl(\mu)) \leq \mu$. 
3. a fuzzy semi-pre open set [9] if $\mu \leq cl(\text{int}(cl(\mu)))$ and a fuzzy semi-pre closed if $\text{int}(\text{int}(\mu)) \leq \mu$. 
4. a fuzzy regular open set [2] if $\mu = \text{int}(cl(\mu))$ and a fuzzy regular closed set if $\mu = cl(\text{int}(\mu))$. 
5. a fuzzy generalized open (fg-open) set [1] if $\omega \geq \text{int}(\text{int}(\mu)) \setminus \text{int}(\text{int}(\mu))$ and a fuzzy generalized closed set if $\omega \leq \text{int}(\text{int}(\mu)) \setminus \text{int}(\text{int}(\mu))$. 
6. a fuzzy b-open (fb-open) set [2] iff $\mu \leq \omega \cup \text{int}(\text{int}(\mu))$ and a fuzzy b-closed set iff $\mu \leq \omega \cap \text{int}(\text{int}(\mu))$.
7. a fuzzy t-set [10] if $\mu = \text{int}(\mu)$.
8. a fuzzy B-set [10] if $\mu = \mu_a \land \mu_b$, where $\mu_a$ is fuzzy open and $\mu_b$ is a fuzzy t-set.
a. The class of all fuzzy pre-open (resp. fuzzy semi-open, fuzzy semi-pre open, fuzzy regular open, fuzzy generalized open, fuzzy b-open, fuzzy t-set and fuzzy B set) sets in a fuzzy topological space $(X, T)$ is denoted by $\text{FPO}(X)$ (resp. $\text{FSO}(X)$, $\text{FSPO}(X)$, $\text{FRO}(X)$, $\text{FGO}(X)$, $\text{FbO}(X)$, $\text{Ft}(X)$ and $\text{FB}(X)$).

**Definition 1.4** [7]

A subset $B$ of a space $X$ is a $\Lambda$-set if $B = B^\wedge$, where $B^\wedge = \bigwedge \{U, U > B, U \in T\}$.

**Definition 1.5** [7]

A subset $A$ of a space $X$ is called $\lambda$-closed if $A = B \land C$, where $B$ is a $\Lambda$-set and $C$ is a closed set.

**Definition 1.6** [7]

A subset $A$ of a space $X$ is called $\lambda$-open if $A^\epsilon = X \setminus A$ is $\lambda$-closed.

**Definition 1.7**[9]

Let $\mu$ be a fuzzy set in a fts $X$. Then fuzzy semi pre-interior of $\mu$ is denoted and defined by $\text{spint}(\mu) = \{\omega: \omega \leq \mu, \mu \in \text{FSPO}(X)\}$.

**Definition 1.8**[8]

A fuzzy set $\mu$ of a fts $X$ is called a fuzzy pre semi open set if $\text{spint}(\mu) \geq \omega$, whenever $\mu \geq \omega$ and $\omega$ is a fuzzy g-closed set in $(X, T)$. The class of all fuzzy pre semi open sets is denoted by $\text{FPSO}(X)$. The complement of a fuzzy pre semi open set is called a fuzzy pre semi closed set in $(X, T)$.

**Definition 1.9** [5]

Let $X$ be a topological space with a topology $T_X$ and $T_Q$ be a subset of $T_X$, i.e., $T_Q$ consists of open sets from $X$.

A scale or discontinuity structure $Q_x = Q$ on $X$ is a mapping $X \to 2^\tau_Q$ that satisfies the following conditions:

- **(SC1)** For all points $x$ from $X$, if $A$ is an element of $Q(x)$, then $x \in A$.
- **(SC2)** Any set from $T_Q$ belongs to some set from $Q(x)$.
2. Fuzzy Pre semi-\(\lambda\) closed sets in fuzzy scalable structure space

**Definition 2.1**

Let \(X\) be a non empty set, \(\tau_i : i \in I\) be the fuzzy topologies associated with \(X\) and let \(\tau^*_X\) be the collection of all fuzzy open sets of \((X, \tau_i)\) and a function \(Q_X : X \to \tau^*_X\) be the fuzzy scale.

A fuzzy scalable structure is denoted by \(Q^*_X\) and defined by \(Q^*_X = \{0,1\} \cup Q^*_X\) where \(Q^*_X = \{Q_X(x) : x \in X\}\).

**Definition 2.2**

Let \(X\) be a non empty set, \(\tau^*_X\) be the collection of all fuzzy open sets of \(X\) and \(Q^*_X\) be a fuzzy scalable structure on \(X\). Then, the triad \((X, \tau^*_X, Q^*_X)\) is called a fuzzy scalable structure space.

Every member of \(Q^*_X\) is called a fuzzy \(Q^*_X\) open set (briefly, \(FQ^*_X\) open). Its complement is said to be a fuzzy \(Q^*_X\) closed set (briefly, \(FQ^*_X\) closed).

**Definition 2.3**

Let \((X, \tau^*_X, Q^*_X)\) be a fuzzy scalable structure space. The fuzzy \(Q^*_X\) interior and fuzzy \(Q^*_X\) closure of a fuzzy set \(\mu\) of \(X\) are defined and denoted as follows:

(i) \[FQ^*_X \text{ int}(\mu) = \bigvee \{\omega : \omega \text{ is a } FQ^*_X \text{ open set and } \omega \leq \mu\}\]

(ii) \[FQ^*_X \text{ cl}(\mu) = \bigwedge \{\omega : \omega \text{ is a } FQ^*_X \text{ closed set and } \omega \geq \mu\}\]

**Definition 2.4**

A fuzzy subset \(\mu\) of a fuzzy scalable structure space \((X, \tau^*_X, Q^*_X)\) is called:

1. A fuzzy \(Q^*_X\) pre-open (in short, \(FQ^*_X\) pre-open) set, if \(\mu \leq FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu))\) and a fuzzy \(Q^*_X\) pre-closed (in short, \(FQ^*_X\) pre-closed) set if \(FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu)) \leq \mu\).

2. A fuzzy \(Q^*_X\) semi-open (in short, \(FQ^*_X\) semi-open) set if \(\mu \leq FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu))\) and a fuzzy \(Q^*_X\) semi-closed (in short, \(FQ^*_X\) semi-closed) set if \(FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu)) \leq \mu\).

3. A fuzzy \(Q^*_X\) semi-pre open (in short, \(FQ^*_X\) sp-open) set if \(\mu \leq FQ^*_X \text{ cl}(FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu)))\) and a fuzzy \(Q^*_X\) semi-pre closed (in short, \(FQ^*_X\) sp-closed) set if \(FQ^*_X \text{ int}(FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu))) \leq \mu\).

4. A fuzzy \(Q^*_X\) regular open (in short, \(FQ^*_X\) regular-open) set if \(\mu = FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu))\) and a fuzzy \(Q^*_X\) regular closed (in short, \(FQ^*_X\) regular-closed) set if \(\mu = FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu))\).

5. A fuzzy \(Q^*_X\) generalized open (in short, \(FQ^*_X\) g-open) set if \(FQ^*_X \text{ int}(\mu) \leq \omega\) whenever \(\mu \leq \omega\) and \(\omega\) is fuzzy \(Q^*_X\) open set in \((X, Q^*_X)\). The complement of fuzzy \(Q^*_X\) g-open set is called fuzzy \(Q^*_X\) g-closed (in short, \(FQ^*_X\) g-closed) set in \((X, \tau^*_X, Q^*_X)\).

6. A fuzzy \(Q^*_X\) b-open (in short, \(FQ^*_X\) b-open) set if \(\mu \leq (FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu)) \lor (FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu)))\) and a fuzzy \(Q^*_X\) b-closed (in short, \(FQ^*_X\) b-closed) set if \(\mu \geq (FQ^*_X \text{ cl}(FQ^*_X \text{ int}(\mu)) \land (FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu)))\).

7. A fuzzy \(Q^*_X\) t-set (in short, \(FQ^*_X\) t-set) if \(FQ^*_X \text{ int}(\mu) = FQ^*_X \text{ int}(FQ^*_X \text{ cl}(\mu))\).

8. A fuzzy \(Q^*_X\) B-set (in short, \(FQ^*_X\) B-set) if \(\mu = \mu_1 \land \mu_2\) where \(\mu_1\) is a \(FQ^*_X\) t-set and \(\mu_2\) is a \(FQ^*_X\) open set.

The class of all \(FQ^*_X\) pre-open (resp. \(FQ^*_X\) semi-open, \(FQ^*_X\) semi-pre open, \(FQ^*_X\) regular open, \(FQ^*_X\) generalized open, \(FQ^*_X\) b open, \(FQ^*_X\) t-set and \(FQ^*_X\) B set) sets in a fuzzy scalable structure space \((X, \tau^*_X, Q^*_X)\) is denoted by \(FQ^*_X\) PO(X) (respectively \(FQ^*_X\) SO(X), \(FQ^*_X\) SPO(X), \(FQ^*_X\) RO(X), \(FQ^*_X\) GO(X), \(FQ^*_X\) bO(X), \(FQ^*_X\) t(X) and \(FQ^*_X\) B(X)).
Definition 2.5
A fuzzy subset \( \mu \) of a fuzzy scalable structure space \((X, \tau'_X, Q^+_X)\) is called:

1. a fuzzy \( Q^+_X \)-\( \Lambda \)-set (in short, \( FQ^+_X \)-\( \Lambda \)-set) if \( \mu = FQ^+_X \mu^\wedge \), where \( FQ^+_X \mu^\wedge = \bigwedge \{ \omega, \omega > \mu, \omega \in Q^+_X \} \).
2. a fuzzy \( Q^+_X \) pre semi -\( \Lambda \)-set (in short, \( FQ^+_X \) pre semi -\( \Lambda \)-set) if \( \mu = FQ^+_X \mu^\wedge \), where \( FQ^+_X PS \mu^\wedge = \bigwedge \{ \omega, \omega > \mu, \omega \in FQ^+_X PS \} \).
3. a fuzzy \( Q^+_X \) regular -\( \Lambda \)-set (in short, \( FQ^+_X \) regular -\( \Lambda \)-set) if \( \mu = FQ^+_X \mu^\wedge \), where \( FQ^+_X \mu^\wedge = \bigwedge \{ \omega, \omega > \mu, \omega \in FQ^+_X PS \} \).
4. a fuzzy \( Q^+_X \) b -\( \Lambda \)-set (in short, \( FQ^+_X \) b -\( \Lambda \)-set) if \( \mu = FQ^+_X b \mu^\wedge \), where \( FQ^+_X b \mu^\wedge = \bigwedge \{ \omega, \omega > \mu, \omega \in FQ^+_X b \} \).

Definition 2.6
A fuzzy subset \( \mu \) of a fuzzy scalable structure space \((X, \tau'_X, Q^+_X)\) is called:

1. a fuzzy \( Q^+_X \) \( \lambda \)-closed (in short, \( FQ^+_X \) \( \lambda \)-closed) if \( \mu = \mu_1 \wedge \mu_2 \), where \( \mu_1 \) is a \( FQ^+_X \) \( \Lambda \)-set and \( \mu_2 \) is a \( FQ^+_X \) closed set.
2. a fuzzy \( Q^+_X \) pre semi \( \lambda \)-closed (in short, \( FQ^+_X \) pre semi \( \lambda \)-closed) if \( \mu = \mu_1 \wedge \mu_2 \), where \( \mu_1 \) is a \( FQ^+_X \) pre semi \( \Lambda \)-set and \( \mu_2 \) is a \( FQ^+_X \) closed set.
3. a fuzzy \( Q^+_X \) regular \( \lambda \)-closed (in short, \( FQ^+_X \) regular \( \lambda \)-closed) if \( \mu = \mu_1 \wedge \mu_2 \), where \( \mu_1 \) is a \( FQ^+_X \) regular \( \lambda \)-set and \( \mu_2 \) is a \( FQ^+_X \) closed set.
4. a fuzzy \( Q^+_X \) b -\( \lambda \) closed (in short, \( FQ^+_X \) b -\( \lambda \) closed) if \( \mu = \mu_1 \wedge \mu_2 \), where \( \mu_1 \) is a \( FQ^+_X \) b -\( \lambda \)-set and \( \mu_2 \) is a \( FQ^+_X \) closed set.

The complement of a \( FQ^+_X \) \( \lambda \)-closed (resp. \( FQ^+_X \) pre semi \( \lambda \)-closed, \( FQ^+_X \) regular \( \lambda \)-closed and \( FQ^+_X \) b -\( \lambda \) closed) set is called a \( FQ^+_X \) \( \lambda \)-open (resp. \( FQ^+_X \) pre semi \( \lambda \) open, \( FQ^+_X \) regular \( \lambda \) open and \( FQ^+_X \) b -\( \lambda \) open).

Proposition 2.1
Every \( FQ^+_X \) semi pre closed set in fuzzy scalable structure space \((X, \tau'_X, Q^+_X)\) is a \( FQ^+_X \) semi pre closed set.

Remark 2.1
The converse part of the above Proposition 2.1 need not be true as shown in the following example.

Example 2.1
Let \( X = \{a, b\} \) be a non empty set and \( \tau'_X \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{0,1, \mu_1, \mu_2\} \) and \( \tau_2 = \{0,1, \mu_1\} \) be the fuzzy topologies on \( X \) where \( \mu_1, \mu_2, \mu_1 \in I^X \) defined as \( \mu_1(a) = 0.7, \mu_1(b) = 0.6; \mu_2(a) = 0.3, \mu_2(b) = 0.2; \mu_1(a) = 0.5, \mu(b) = 0.4 \). Now, define \( Q_X : X \rightarrow \tau'_X \) by \( Q_X(a) = \mu_1; Q_X(b) = \mu_2 \). Then, \( Q_X \) is a fuzzy scale and \( Q^+_X = \{ \mu_1, \mu_2 \} \). Let \( Q^+_X = \{0,1, \mu_1, \mu_3\} \) be a fuzzy scalable structure on \( X \). Clearly, the triad \((X, \tau'_X, Q^+_X)\) is fuzzy scalable structure space. Here the fuzzy set \( \mu_2 \) is a \( FQ^+_X \) semi pre closed but not \( FQ^+_X \) closed.

Proposition 2.2
Every \( FQ^+_X \) semi pre closed set in fuzzy scalable structure space \((X, \tau'_X, Q^+_X)\) is a \( FQ^+_X \) semi closed set.

Remark 2.2
The converse part of the above Proposition 2.2 need not be true as shown in the following example.
Example 2.2

Let \( X = \{a, b\} \) be a non empty set and \( \tau^*_x \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{0.1, \mu, \mu_1\} \) and \( \tau_2 = \{0.1, \mu_1\} \) be the fuzzy topologies on \( X \) where \( \mu, \mu_2, \mu_3 \in \mathbb{R}^+ \) are defined as \( \mu(a) = 0.2, \mu(b) = 0.3; \ \mu_2(a) = 0.7, \mu_2(b) = 0.7; \ \mu_3(a) = 0.3, \mu_3(b) = 0.4 \). Now, define \( Q_x : X \to \tau^*_x \) by \( Q_x(a) = \mu_1; \ Q_x(b) = \mu_3 \). Then, \( Q_x \) is a fuzzy scale and \( Q^*_x = \{\mu_1, \mu_3\} \). Let \( \tau^*_x = \{\mu_1, \mu_3\} \) be a fuzzy scale structure on \( X \). Clearly, the triad \((X, \tau^*_x, Q^*_x)\) is fuzzy scale structure space. Here the fuzzy set \( \mu_2 \) is a \( FQ^*_x \) pre semi closed but not \( FQ^*_x \) semi closed.

Proposition 2.3

Every \( FQ^*_x \) closed set in fuzzy scale structure space \((X, \tau^*_x, Q^*_x)\) is a \( FQ^*_x \) pre semi closed set.

Remark 2.3

The converse part of the above Proposition 2.3 need not be true as shown in the following example.

Example 2.3

In example 2.1, the fuzzy set \( \mu_2 \) is a \( FQ^*_x \) pre semi closed but not \( FQ^*_x \) closed.

Proposition 2.4

Every \( FQ^*_x \) regular closed set in fuzzy scale structure space \((X, \tau^*_x, Q^*_x)\) is a \( FQ^*_x \) semi pre closed set.

Remark 2.4

The converse part of the above Proposition 2.4 need not be true as shown in the following example.

Example 2.4

In example 2.1, the fuzzy set \( \mu_2 \) is a \( FQ^*_x \) semi pre closed but not \( FQ^*_x \) regular closed.

Proposition 2.5

Every \( FQ^*_x \) regular closed set in fuzzy scale structure space \((X, \tau^*_x, Q^*_x)\) is a \( FQ^*_x \) pre semi closed set.

Remark 2.5

The converse part of the above Proposition 2.5 need not be true as shown in the following example.

Example 2.5

In example 2.1, the fuzzy set \( \mu_2 \) is \( FQ^*_x \) pre semi closed but not \( FQ^*_x \) regular closed.

Proposition 2.6

Every \( FQ^*_x \) \( b \)-closed set in fuzzy scale structure space \((X, \tau^*_x, Q^*_x)\) is a \( FQ^*_x \) semi pre closed set.

Remark 2.6

The converse part of the above Proposition 2.6 need not be true as shown in the following example.

Example 2.6

In example 2.1, the fuzzy set \( \mu_2 \) is \( FQ^*_x \) semi pre closed but not \( FQ^*_x \) \( b \)-closed.

Proposition 2.7

Every \( FQ^*_x \) \( b \)-closed set in fuzzy scale structure space \((X, \tau^*_x, Q^*_x)\) is a \( FQ^*_x \) pre semi closed set.

Remark 2.7

The converse part of the above Proposition 2.7 need not be true as shown in the following example.
Example 2.7

In example 2.1, the fuzzy set \( \mu_2 \) is a \( FQ_X^w \) pre semi closed but not \( FQ_X^w \)-b-closed.

Proposition 2.8

Every \( FQ_X^w \) closed set in fuzzy scalable structure space \((X, \tau_X^*, Q_X^w)\) is:

1. \( FQ_X^w \) pre semi- \( \lambda \) closed set
2. \( FQ_X^w \)-t-set
3. \( FQ_X^w \)-B-set
4. \( FQ_X^w \)-b- \( \lambda \) closed set
5. \( FQ_X^w \)- \( \lambda \) closed set
6. \( FQ_X^w \)-regular- \( \lambda \) closed set

Remark 2.8

The converse part of the above Proposition 2.8 need not be true as shown in the following examples.

Example 2.8

Let \( X = \{ a, b \} \) be a non empty set and \( \tau_X^* \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{ 0,3 \}, \mu_1, \mu_2, \mu_3, \mu_4 \} \) and \( \tau_2 = \{ 0,3 \} \) be the fuzzy topologies on \( X \) where \( \mu_1, \mu_2, \mu_3, \mu_4 \) \( \in I_X \) defined as \( \mu_1(a) = 0.4, \mu_2(a) = 0.5; \mu_3(a) = 0.4, \mu_4(a) = 0.1; \mu_1(b) = 0.1, \mu_2(b) = 0.8, \mu_3(b) = 0.5, \mu_4(b) = 0.3 \). Now, define \( Q_X : X \rightarrow \tau_X^* \) by \( Q_X(a) = \mu_1; Q_X(b) = \mu_3 \). Then, \( Q_X \) is a fuzzy scale and \( \omega \) is a fuzzy scalable structure on \( X \). Clearly, the triad \((X, \tau_X^*, Q_X^w)\) is fuzzy scalable structure space. Here the fuzzy set \( \omega \) defined by \( a_0 = 0.4, a_b = 0.4 \) is a \( FQ_X^w \) pre semi- \( \lambda \) closed, \( FQ_X^w \)-t-set, \( FQ_X^w \)-B-set and \( FQ_X^w \)-b- \( \lambda \) set but not \( FQ_X^w \)-closed set. The fuzzy set \( \mu_2 \) is \( FQ_X^w \)- \( \lambda \) closed but not \( FQ_X^w \)-closed.

Example 2.9

Let \( X = \{ a, b \} \) be a non empty set and \( \tau_X^* \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{ 0,3 \}, \mu_1, \mu_2, \mu_3, \mu_4 \} \) and \( \tau_2 = \{ 0,3 \} \) be the fuzzy topologies on \( X \) where \( \mu_1, \mu_2, \mu_3, \mu_4 \) \( \in I_X \) defined as \( \mu_1(a) = 0.4, \mu_2(a) = 0.5; \mu_3(a) = 0.4, \mu_4(a) = 0.1; \mu_1(b) = 0.1, \mu_2(b) = 0.8, \mu_3(b) = 0.5, \mu_4(b) = 0.3 \) . Now, define \( Q_X : X \rightarrow \tau_X^* \) by \( Q_X(a) = \mu_1; Q_X(b) = \mu_3 \). Then, \( Q_X \) is a fuzzy scale and \( \omega \) is a fuzzy scalable structure on \( X \). Clearly, the triad \((X, \tau_X^*, Q_X^w)\) is fuzzy scalable structure space. Here the fuzzy set \( \omega \) defined by \( a_0 = 0.1, a_b = 0.4 \) is a \( FQ_X^w \)-regular- \( \lambda \) closed but not \( FQ_X^w \)-closed.

Proposition 2.9

Every \( FQ_X^w \)- \( \lambda \) closed set is a \( FQ_X^w \)-pre semi- \( \lambda \) closed set in fuzzy scalable structure space \((X, \tau_X^*, Q_X^w)\).

Remark 2.9

The converse part of the above Proposition 2.9 need not be true as shown in the following example.

Example 2.10

In example 2.8 the fuzzy set \( \mu_4 \) is a \( FQ_X^w \) pre semi- \( \lambda \) closed but not \( FQ_X^w \)- \( \lambda \) closed set in \( (X, \tau_X^*, Q_X^w) \).

Proposition 2.10

Every \( FQ_X^w \)-regular- \( \lambda \) closed set is a \( FQ_X^w \)-pre semi- \( \lambda \) closed set in fuzzy scalable structure space \((X, \tau_X^*, Q_X^w)\).
Remark 2.10
The converse part of the above Proposition 2.10 need not be true as shown in the example.

Example 2.11
In example 2.8 the fuzzy set \( \mu \) is a \( FQ^+ \) pre-\( \lambda \) closed but not \( FQ^+ \) regular - \( \lambda \) closed set in \( (X, \tau^+ \lambda, Q^+ \lambda) \).

Proposition 2.11
Every \( FQ^+ \) b - \( \lambda \) closed set is a \( FQ^+ \) pre-\( \lambda \) closed set in fuzzy scalable structure space \( (X, \tau^+ \lambda, Q^+ \lambda) \).

Remark 2.11
The converse part of the above Proposition 2.11 need not be true as shown in the following example.

Example 2.12
In example 2.8 the fuzzy set \( \omega \) defined by \( \omega(a) = 0, \omega(b) = 0.6 \) is a \( FQ^+ \) pre-\( \lambda \) closed but not \( FQ^+ \) b - \( \lambda \) closed set in \( (X, \tau^+ \lambda, Q^+ \lambda) \).

Proposition 2.12
Every \( FQ^+ \) \( t \)-set is a \( FQ^+ \) \( B \)-set in the fuzzy scalable structure space \( (X, \tau^+ \lambda, Q^+ \lambda) \).

Remark 2.12
The converse part of the above Proposition 2.12 need not be true as shown in the following example.

Example 2.13
Let \( X = \{ a, b \} \) be a non empty set and \( \tau^+ \lambda \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{ 0, 1, \mu_1, \mu_2 \} \) and \( \tau_2 = \{ 0, 1, \mu_2, \mu_3 \} \) be the fuzzy topologies on \( X \) where \( \mu_1, \mu_2, \mu_3 \in I^X \) defined as \( \mu_1(a) = 0.2, \mu_1(b) = 0.3; \mu_2(a) = 0.5, \mu_2(b) = 0.6 \) and \( \mu_3(a) = 0.1, \mu_3(b) = 0.2 \). Now, define \( Q_x : X \rightarrow \tau^+ \lambda \) by \( Q_x(a) = \mu_1; Q_x(b) = \mu_2 \). Then, \( Q_x \) is a fuzzy scale and \( Q_x = \{ \mu_1 \} \). Let \( Q^+ \lambda = \{ 0, 1, \mu_2, \mu_3 \} \) be a fuzzy scalable structure on \( X \). Clearly, the triad \( (X, \tau^+ \lambda, Q^+ \lambda) \) is a fuzzy scalable structure space. Here the fuzzy set \( \mu_2 \) is a \( FQ^+ \) \( B \)-set but not a \( FQ^+ \) \( t \)-set.

Proposition 2.13
Every \( FQ^+ \) regular - \( \lambda \) closed set is a \( FQ^+ \) \( t \)-set in the fuzzy scalable structure space \( (X, \tau^+ \lambda, Q^+ \lambda) \).

Remark 2.13
The converse part of the above Proposition 2.13 need not be true as shown in the following example.

Example 2.14
Let \( X = \{ a, b \} \) be a non empty set and \( \tau^+ \lambda \) be the collection of all fuzzy open sets in \( X \). Let \( \tau_1 = \{ 0, 1, \mu_1, \mu_2 \} \) and \( \tau_2 = \{ 0, 1, \mu_2 \} \) be the fuzzy topologies on \( X \) where \( \mu_1, \mu_2, \mu_3 \in I^X \) are defined as \( \mu_1(a) = 0.6, \mu_1(b) = 0.2; \mu_2(a) = 0.7, \mu_2(b) = 0.2 \). Now define \( Q_x : X \rightarrow \tau^+ \lambda \) by \( Q_x(a) = \mu_2; Q_x(b) = \mu_3 \). Then, \( Q_x \) is a fuzzy scale and \( Q_x = \{ \mu_2, \mu_3 \} \). Let \( Q^+ \lambda = \{ 0, 1, \mu_2, \mu_3 \} \) be a fuzzy scalable structure on \( X \). Clearly, the triad \( (X, \tau^+ \lambda, Q^+ \lambda) \) is a fuzzy scalable structure space. Here the fuzzy set \( \omega \) defined by \( \omega(a) = 0.8, \omega(b) = 0.2 \) is a \( FQ^+ \) \( t \)-set but not a \( FQ^+ \) regular closed set.
Proposition 2.14
Every $FQ^n_X$ regular - $\lambda$ closed set is a $FQ^n_X$ b - $\lambda$ closed set in the fuzzy scalable structure space $(X, \tau^*_X, Q^n_X)$.

Remark 2.14
The converse part of the above Proposition 2.14 need not be true as shown in the following example.

Example 2.15
In example 2.14, the fuzzy set $\omega$ defined by $\omega(a) = 0.3, \omega(b) = 0.2$ is a $FQ^n_X$ b - $\lambda$ closed set but not a $FQ^n_X$ regular - $\lambda$ closed set.

Proposition 2.15
Every $FQ^n_X$ $\lambda$ - closed set is a $FQ^n_X$ b - $\lambda$ closed set in the fuzzy scalable structure space $(X, \tau^*_X, Q^n_X)$.

Remark 2.15
The converse part of the above Proposition 2.15 need not be true as shown in the following example.

Example 2.16
In example 2.14, the fuzzy set $\omega$ defined by $\omega(a) = 0.8, \omega(b) = 0.2$ is a $FQ^n_X$ b - $\lambda$ closed set but not a $FQ^n_X$ regular - $\lambda$ closed set.

Remark 2.16
From the above discussion we have the following implications are true:

Definition 3.1
Let $(X_1, \tau^*_{X_1}, Q^n_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^n_{X_2})$ be any two fuzzy scalable structure spaces. Then the function $f : (X_1, \tau^*_{X_1}, Q^n_{X_1}) \rightarrow (X_2, \tau^*_{X_2}, Q^n_{X_2})$ is said to be $FQ^n_X$ continuous if for every $FQ^n_{X_2}$ - closed set in $(X_2, \tau^*_{X_2}, Q^n_{X_2})$ there exists a $FQ^n_{X_1}$ - closed set in $(X_1, \tau^*_{X_1}, Q^n_{X_1})$.

Definition 3.2
Let $(X_1, \tau^*_{X_1}, Q^n_{X_1})$ and $(X_2, \tau^*_{X_2}, Q^n_{X_2})$ be any two fuzzy scalable structure spaces. Then the function $f : (X_1, \tau^*_{X_1}, Q^n_{X_1}) \rightarrow (X_2, \tau^*_{X_2}, Q^n_{X_2})$ is said to be:
1. $FQ^n_X$ pre semi- $\lambda$ continuous if for every $FQ^n_{X_2}$ - closed set in $(X_2, \tau^*_{X_2}, Q^n_{X_2})$ there exists a $FQ^n_{X_1}$ pre semi- $\lambda$ - closed set in $(X_1, \tau^*_{X_1}, Q^n_{X_1})$. 

Figure: 1
Relationship between $FQ^n_X$ - pre semi- $\lambda$ closed set and other existing $FQ^n_X$ - closed sets

3. $FQ^n_X$ Pre semi- $\lambda$ continuity in fuzzy scalable structure spaces

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2. \( FQ_{X_1}^* \) t-continuous if for every \( FQ_{X_1}^* \) closed set in \( (X_2, \tau_{X_2}, Q_{X_2}^*) \) there exists a \( FQ_{X_1}^* \) t-closed set in \( (X_1, \tau_{X_1}, Q_{X_1}^*) \).

3. \( FQ_{X_1}^* \) B-continuous if for every \( FQ_{X_1}^* \) closed set in \( (X_2, \tau_{X_2}, Q_{X_2}^*) \) there exists a \( FQ_{X_1}^* \) B-closed set in \( (X_1, \tau_{X_1}, Q_{X_1}^*) \).

4. \( FQ_{X_1}^* \) b-\( \lambda \) continuous if for every \( FQ_{X_1}^* \) closed set in \( (X_2, \tau_{X_2}, Q_{X_2}^*) \) there exists a \( FQ_{X_1}^* \) b-\( \lambda \) closed set in \( (X_1, \tau_{X_1}, Q_{X_1}^*) \).

5. \( FQ_{X_1}^* \) \( \lambda \)-continuous if for every \( FQ_{X_1}^* \) closed set in \( (X_2, \tau_{X_2}, Q_{X_2}^*) \) there exists a \( FQ_{X_1}^* \) \( \lambda \)-closed set in \( (X_1, \tau_{X_1}, Q_{X_1}^*) \).

6. \( FQ_{X_1}^* \) regular-\( \lambda \)-continuous if for every \( FQ_{X_1}^* \) closed set in \( (X_2, \tau_{X_2}, Q_{X_2}^*) \) there exists a fuzzy \( FQ_{X_1}^* \) regular-\( \lambda \)-closed set in \( (X_1, \tau_{X_1}, Q_{X_1}^*) \).

**Proposition 3.1**

Let \((X_1, \tau_{X_1}, Q_{X_1}^*)\) and \((X_2, \tau_{X_2}, Q_{X_2}^*)\) be any two fuzzy scalable structure spaces. If a function \( f : (X_1, \tau_{X_1}, Q_{X_1}^*) \rightarrow (X_2, \tau_{X_2}, Q_{X_2}^*) \) is \( FQ_{X}^* \) continuous function then it is:

1. a \( FQ_{X}^* \) pre semi-\( \lambda \)-continuous function.
2. a \( FQ_{X}^* \) t-continuous function.
3. a \( FQ_{X}^* \) B-continuous function.
4. a \( FQ_{X}^* \) b-\( \lambda \)-continuous function.
5. a \( FQ_{X}^* \) \( \lambda \)-continuous function.
6. a \( FQ_{X}^* \) regular-\( \lambda \)-continuous function.

**Remark 3.1**

The converse part of the above Proposition 3.1 need not be true as shown in the following examples.

**Example 3.1**

Let \( X_1 = \{a, b\} \) be a non empty set and \( \tau_{X_1}^* \) be the collection of all fuzzy open sets in \( X_1 \). Let \( \tau_1 = \{0,1, \mu_1, \mu_2, \mu_3, \mu_4, \mu_5\} \) and \( \tau_2 = \{0,1, \mu_4\} \) be the fuzzy topologies on \( X_1 \) where \( \mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \in I^{X_1} \) are defined as \( \mu_1(a) = 0.4, \mu_1(b) = 0.3, \mu_2(a) = 0.4, \mu_2(b) = 0.1, \mu_3(a) = 0.8, \mu_3(b) = 0.1; \mu_4(a) = 0.3, \mu_4(b) = 0.3; \mu_5(a) = 0.3, \mu_5(b) = 0.1. \) Now, define \( Q_{X_1}^* : X_1 \rightarrow \tau_{X_1}^* \) by \( Q_{X_1}^* : X_1 \rightarrow \tau_{X_1}^* \) by \( Q_{X_1}^*(a) = \mu_1; Q_{X_1}^*(b) = \mu_2. \) Then, \( Q_{X_1}^* \) is a fuzzy scale and \( Q_{X_1}^* = \{\mu_1, \mu_2\}. \) Let \( Q_{X_1}^* = \{0,1, \mu_4, \mu_5\} \) be a fuzzy scalable structure on \( X_1 \). Clearly, the triad \((X_1, \tau_{X_1}^*, Q_{X_1}^*)\) is a fuzzy scalable structure space.

Let \( X_2 = \{u, v\} \) be a non empty set and \( \tau_{X_2}^* \) be the collection of all fuzzy open sets in \( X_2 \). Let \( \tau_3 = \{0,1, \omega_1, \omega_2\} \) and \( \tau_4 = \{0,1, \omega_4\} \) be the fuzzy topologies on \( X_2 \) where \( \omega_1, \omega_2, \omega_3 \in I^{X_2} \) are defined as \( \omega_1(u) = 0.2, \omega_1(v) = 0.2; \omega_2(u) = 0.4, \omega_2(v) = 0.3; \omega_3(u) = 0.6, \omega_3(v) = 0.7. \) Now, define \( Q_{X_2}^* : X_2 \rightarrow \tau_{X_2}^* \) by \( Q_{X_2}^*(u) = \omega_1; Q_{X_2}^*(v) = \omega_2. \) Then, \( Q_{X_2}^* \) is a fuzzy scale and \( Q_{X_2}^* = \{\omega_1, \omega_2\}. \) Let \( Q_{X_2}^* = \{0,1, \omega_1, \omega_3\} \) be a fuzzy scalable structure on \( X_2 \). Clearly, the triad \((X_2, \tau_{X_2}^*, Q_{X_2}^*)\) is a fuzzy scalable structure space.

Let \( f : (X_1, \tau_{X_1}^*, Q_{X_1}^*) \rightarrow (X_2, \tau_{X_2}^*, Q_{X_2}^*) \) be a \( FQ_{X}^* \) continuous function defined by \( f(a) = u \) and \( f(b) = v. \) Then the function ‘\( f \)’ is \( FQ_{X}^* \) pre semi-\( \lambda \)-continuous and \( FQ_{X}^* \) b-\( \lambda \)-continuous but not \( FQ_{X}^* \) continuous.
Example 3.2
Let $X = \{a, b\}$ be a non empty set and $\tau_X$ be the collection of all fuzzy open sets in $X$. Let $\tau_1 = \{0, 1, \mu_1, \mu_2\} \quad \text{and} \quad \tau_2 = \{0, 1, \mu_3\}$ be the fuzzy topologies on $X_1$ where $\mu_1, \mu_2, \mu_3 \in F^X_1$ are defined as $\mu_1(a) = 0.6, \mu_1(b) = 0.2; \quad \mu_2(a) = 0.7, \mu_2(b) = 0.2; \quad \mu_3(a) = 0.1, \quad \mu_3(b) = 0.8$. Now, define $Q_{X_1} : X_1 \rightarrow \tau_{X_1}$ by $Q_{X_1}(a) = \mu_2; \quad Q_{X_1}(b) = \mu_1$. Then, $Q_{X_1}$ is a fuzzy scale and $Q_{X_1}^{\ast} = \{\mu_2, \mu_3\}$. Let $Q_{X_1}^\ast = \{0, 1, \mu_2, \mu_3\}$ be a fuzzy scalable structure on $X_1$. Clearly, the triad $(X_1, \tau_{X_1}, Q_{X_1}^\ast)$ is a fuzzy scalable structure space.

Example 3.3
Let $X = \{a, b\}$ be a non empty set and $\tau_X$ be the collection of all fuzzy open sets in $X$. Let $\tau_1 = \{0, 1, \omega_1, \omega_2\} \quad \text{and} \quad \tau_2 = \{0, 1, \omega_3\}$ be the fuzzy topologies on $X_2$ where $\omega_1, \omega_2, \omega_3 \in F^X_2$ are defined as $\omega_1(a) = 0.3, \omega_1(v) = 0.8; \quad \omega_2(u) = 0.2, \omega_2(v) = 0.3; \quad \omega_3(u) = 0.6, \quad \omega_3(v) = 0.7$. Now, define $Q_{X_2} : X_2 \rightarrow \tau_{X_2}$ by $Q_{X_2}(u) = \omega_2; \quad Q_{X_2}(v) = \omega_1$. Then, $Q_{X_2}$ is a fuzzy scale and $Q_{X_2}^{\ast} = \{\omega_1\}$. Let $Q_{X_2}^\ast = \{0, 1, \omega_1\}$ be a fuzzy scalable structure on $X_2$. Clearly, the triad $(X_2, \tau_{X_2}, Q_{X_2}^\ast)$ is a fuzzy scalable structure space.

Example 3.4
Let $X = \{a, b\}$ be a non empty set and $\tau_X$ be the collection of all fuzzy open sets in $X$. Let $\tau_1 = \{0, 1, \mu_1, \mu_2, \mu_3, \mu_4\} \quad \text{and} \quad \tau_2 = \{0, 1, \mu_1\}$ be the fuzzy topologies on $X_1$ where $\mu_1, \mu_2, \mu_3, \mu_4 \in F^X_1$ are defined as $\mu_1(a) = 0.4, \mu_1(b) = 0.3; \quad \mu_2(a) = 0.4, \mu_2(b) = 0.1; \quad \mu_3(a) = 0.8, \quad \mu_3(b) = 0.1; \quad \mu_4(a) = 0.3, \quad \mu_4(b) = 0.3; \quad \mu_4(a) = 0.3, \mu_4(b) = 0.1$. Now, define $Q_{X_1} : X_1 \rightarrow \tau_{X_1}$ by $Q_{X_1}(a) = \mu_4; \quad Q_{X_1}(b) = \mu_3$. Then, $Q_{X_1}$ is a fuzzy scale and $Q_{X_1}^{\ast} = \{\mu_1, \mu_3\}$. Let $Q_{X_1}^\ast = \{0, 1, \mu_1, \mu_3\}$ be a fuzzy scalable structure on $X_1$. Clearly, the triad $(X_1, \tau_{X_1}, Q_{X_1}^\ast)$ is a fuzzy scalable structure space.
Let \( X_2 = \{ u, v \} \) be a non empty set and \( \tau_{x_2}^* \) be the collection of all fuzzy open sets in \( X_2 \). Let \( \tau_3 = \{ 0.1, \omega_1, \omega_2 \} \) and \( \tau_4 = \{ 0.1, \omega_3 \} \) be the fuzzy topologies on \( X_2 \) where \( \omega_1, \omega_2, \omega_3 \in I^{x_2} \) defined as \\
\( \omega_1(u) = 0.6, \omega_2(v) = 0.9 \); \( \omega_3(u) = 0.4, \omega_3(v) = 0.3 \); \( \omega_4(u) = 0.8, \omega_4(v) = 0.9 \). Now, define \\
\( Q_{x_2} : X_2 \rightarrow \tau_{x_2}^* \) by \( Q_{x_2}(u) = \omega_1; Q_{x_2}(v) = \omega_3 \). Then, \( Q_{x_2} \) is a fuzzy scale and \( Q_{x_2}^* = \{ \omega_1, \omega_3 \} \). Let \( Q_{x_2}^* = \{ 0.1, \omega_1, \omega_3 \} \) be a fuzzy scalable structure on \( X_2 \). Clearly, the triad \( (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) is a fuzzy scalable structure space.

Let \( f : (X_1, \tau_{x_1}^*, Q_{x_1}^*) \rightarrow (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) be a \( FQ_{x_1}^* \) continuous function defined by \\
f(a) = u \text{ and } f(b) = v. \text{ Then the function } f \text{ is } FQ_{x_1}^* \lambda \text{- continuous but not } FQ_{x_1}^* \text{ continuous.}

**Proposition 3.2**

Let \( (X_1, \tau_{x_1}^*, Q_{x_1}^*) \) and \( (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) be any two fuzzy scalable structure spaces. If a function \( f : (X_1, \tau_{x_1}^*, Q_{x_1}^*) \rightarrow (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) is \( FQ_{x_1}^* \lambda \text{- continuous then it is } FQ_{x_1}^* \text{ pre semi-} \lambda \text{ continuous.}

**Remark 3.2**

The converse part of the above Proposition 3.2 need not be true as shown in the following example.

**Example 3.5**

In example 3.1, the function \( f \) is \( FQ_{x_1}^* \) pre semi- \( \lambda \text{- continuous but not } FQ_{x_1}^* \lambda \text{- continuous.} \)

**Proposition 3.3**

Let \( (X_1, \tau_{x_1}^*, Q_{x_1}^*) \) and \( (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) be any two fuzzy scalable structure spaces. If a function \( f : (X_1, \tau_{x_1}^*, Q_{x_1}^*) \rightarrow (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) is \( FQ_{x_1}^* \) regular \( \lambda \text{- continuous then it is } FQ_{x_1}^* \text{ pre semi-} \lambda \text{ continuous.}

**Remark 3.3**

The converse part of the above Proposition 3.3 need not be true as shown in the following example.

**Example 3.6**

In example 3.1, the function \( f \) is \( FQ_{x_1}^* \) pre semi- \( \lambda \text{- continuous but not } FQ_{x_1}^* \) regular - \( \lambda \text{ continuous.}

**Proposition 3.4**

Let \( (X_1, \tau_{x_1}^*, Q_{x_1}^*) \) and \( (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) be any two fuzzy scalable structure spaces. If a function \( f : (X_1, \tau_{x_1}^*, Q_{x_1}^*) \rightarrow (X_2, \tau_{x_2}^*, Q_{x_2}^*) \) is \( FQ_{x_1}^* \) b- \( \lambda \text{- continuous then it is } FQ_{x_1}^* \text{ pre semi-} \lambda \text{ continuous.}

**Remark 3.4**

The converse part of the above Proposition 3.4 need not be true as shown in the following example.

**Example 3.7**

Let \( X_1 = \{ a, b \} \) be a non empty set and \( \tau_{x_1}^* \) be the collection of all fuzzy open sets in \( X_1 \). Let \( \tau_1 = \{ 0.1, \mu_1, \mu_2, \mu_3, \mu_4 \} \) and \( \tau_2 = \{ 0.1, \mu_1 \} \) be the fuzzy topologies on \( X_1 \) where \( \mu_1, \mu_2, \mu_3, \mu_4, \mu_5 \in I^{x_1} \) are defined as \\
\( \mu_1(a) = 0.4, \mu_1(b) = 0.3 \); \( \mu_2(a) = 0.4, \mu_2(b) = 0.1 \); \( \mu_3(a) = 0.8, \mu_3(b) = 0.1 \); \( \mu_4(a) = 0.3, \mu_4(b) = 0.3 \); \( \mu_5(a) = 0.3, \mu_5(b) = 0.1. \) Now, define \( Q_{x_1} : X_1 \rightarrow \tau_{x_1}^* \) by \\
\( Q_{x_1}(a) = \mu_1; Q_{x_1}(b) = \mu_1 \). Then, \( Q_{x_1} \) is a fuzzy scale and \( Q_{x_1}^* = \{ \mu_1 \} \). Let \( Q_{x_1}^* = \{ 0.1, \mu_1, \mu_1 \} \) be a fuzzy scalable structure on \( X_1 \). Clearly, the triad \( (X_1, \tau_{x_1}^*, Q_{x_1}^*) \) is a fuzzy scalable structure space.
Let $X_2=\{u,v\}$ be a non empty set and $\tau_{x_2}$ be the collection of all fuzzy open sets in $X_2$. Let $\tau_3=\{0.1,\omega_1,\omega_2\}$ and $\tau_4=\{0.1,\omega_3\}$ be the fuzzy topologies on $X_2$ where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ defined as $\omega_1(u)=0.2, \omega_1(v)=0.8; \omega_2(u)=0.2, \omega_2(v)=0.3; \omega_3(u)=0.9, \omega_3(v)=0.8$. Now, define $Q_{x_2}: X_2 \rightarrow \tau_{x_2}$ by $Q_{x_2}(u)=\omega_1; Q_{x_2}(v)=\omega_3$. Then, $Q_{x_2}$ is a fuzzy scale and $Q_{x_2}^* = \{\omega_1, \omega_3\}$. Let $Q_{x_2}^* = \{0.1, \omega_1, \omega_3\}$ be a fuzzy scalable structure on $X_2$. Clearly, the triad $(X_2, \tau_{x_2}, Q_{x_2}^-)$ is a fuzzy scalable structure space.

Let $f: (X_1, \tau_{x_1}, Q_{x_1}^-) \rightarrow (X_2, \tau_{x_2}, Q_{x_2}^-)$ be a fuzzy $FQ_x^-$ function defined by $f(a)=u$ and $f(b)=v$. Then the function $f$ is $FQ_x^-$ pre semi-$\lambda$ continuous but not $FQ_x^-$ b- $\lambda$ continuous.

**Proposition 3.5**

Let $(X_1, \tau_{x_1}, Q_{x_1}^-)$ and $(X_2, \tau_{x_2}, Q_{x_2}^-)$ be any two fuzzy scalable structure spaces. If a function $f: (X_1, \tau_{x_1}, Q_{x_1}^-) \rightarrow (X_2, \tau_{x_2}, Q_{x_2}^-)$ is $FQ_x^-$ t- continuous then it is $FQ_x^-$ B -continuous.

**Remark 3.5**

The converse part of the above Proposition 3.5 need not be true as shown in the following example.

**Example 3.8**

In example 2.13, define the mapping $f: (X_1, \tau_{x_1}, Q_{x_1}^-) \rightarrow (X_2, \tau_{x_2}, Q_{x_2}^-)$ by $f(a)=a$ and $f(b)=b$. Then $f$ is $FQ_x^-$ B - continuous but not $FQ_x^-$ t-continuous.

**Proposition 3.6**

Let $(X_1, \tau_{x_1}, Q_{x_1}^-)$ and $(X_2, \tau_{x_2}, Q_{x_2}^-)$ be any two fuzzy scalable structure spaces. If a function $f: (X_1, \tau_{x_1}, Q_{x_1}^-) \rightarrow (X_2, \tau_{x_2}, Q_{x_2}^-)$ is $FQ_x^-$ regular - $\lambda$ continuous then it is $FQ_x^-$ t - continuous.

**Remark 3.6**

The converse part of the above Proposition 3.6 need not be true as shown in the following example.

**Example 3.9**

Let $X_1=\{a,b\}$ be a non empty set and $\tau_{x_1}$ be the collection of all fuzzy open sets in $X_1$. Let $\tau_1=\{0.1,\mu_1,\mu_2\}$ and $\tau_2=\{0.1,\mu_3\}$ be the fuzzy topologies on $X_1$ where $\mu_1,\mu_2,\mu_3 \in I^{X_1}$ are defined as $\mu_1(a)=0.6, \mu_1(b)=0.2; \mu_2(a)=0.7, \mu_2(b)=0.2; \mu_3(a)=0.1, \mu_3(b)=0.8$. Now define $Q_{x_1}: X_1 \rightarrow \tau_{x_1}$ by $Q_{x_1}(a)=\mu_2; Q_{x_1}(b)=\mu_1$. Then, $Q_{x_1}$ is a fuzzy scale and $Q_{x_1}^* = \{\mu_2, \mu_1\}$. Let $Q_{x_1}^* = \{0.1, \mu_2, \mu_1\}$ be a fuzzy scalable structure on $X_1$. Clearly, the triad $(X_1, \tau_{x_1}, Q_{x_1}^-)$ is a fuzzy scalable structure space.

Let $X_2=\{u,v\}$ be a non empty set and $\tau_{x_2}$ be the collection of all fuzzy open sets in $X_2$. Let $\tau_3=\{0.1,\omega_1,\omega_2\}$ and $\tau_4=\{0.1,\omega_3\}$ be the fuzzy topologies on $X_2$ where $\omega_1,\omega_2,\omega_3 \in I^{X_2}$ are defined as $\omega_1(u)=0.2, \omega_1(v)=0.8; \omega_2(u)=0.2, \omega_2(v)=0.3; \omega_3(u)=0.9, \omega_3(v)=0.8$. Now, define $Q_{x_2}: X_2 \rightarrow \tau_{x_2}$ by $Q_{x_2}(u)=\omega_1; Q_{x_2}(v)=\omega_3$. Then, $Q_{x_2}$ is a fuzzy scale and $Q_{x_2}^* = \{\omega_1, \omega_3\}$. Let $Q_{x_2}^* = \{0.1, \omega_1, \omega_3\}$ be a fuzzy scalable structure on $X_2$. Clearly, the triad $(X_2, \tau_{x_2}, Q_{x_2}^-)$ is a fuzzy scalable structure space.

Let $f: (X_1, \tau_{x_1}, Q_{x_1}^-) \rightarrow (X_2, \tau_{x_2}, Q_{x_2}^-)$ be a $FQ_x^-$ continuous function defined by $f(a)=u$ and $f(b)=v$. Then the function $f$ is $FQ_x^-$ t - continuous but not $FQ_x^-$ regular - $\lambda$ continuous.
Proposition 3.7
Let \((X_1, \tau^*_{X_1}, Q^*_{X_1})\) and \((X_2, \tau^*_{X_2}, Q^*_{X_2})\) be any two fuzzy scalable structure spaces. If a function \(f : (X_1, \tau^*_{X_1}, Q^*_{X_1}) \rightarrow (X_2, \tau^*_{X_2}, Q^*_{X_2})\) is \(FQ^*_X\) regular - \(\lambda\) continuous then it is \(FQ^*_X\) b - \(\lambda\) continuous.

Remark 3.7
The converse part of the above Proposition 3.7 need not be true as shown in the following example.

Example 3.10
Let \(X_1=\{a,b\}\) be a non empty set and \(\tau^*_{X_1}\) be the collection of all fuzzy open sets in \(X_1\). Let \(\tau_1=\{0,1,\mu_1,\mu_2\}\) and \(\tau_2=\{0,1,\mu_1\}\) be the fuzzy topologies on \(X_1\) where \(\mu_1, \mu_2, \mu_3 \in \mathbb{T}^X\) are defined as \(\mu_1(a) = 0.6, \mu_2(b) = 0.2\); \(\mu_2(a) = 0.7, \mu_2(b) = 0.2\); \(\mu_3(a) = 0.1, \mu_3(b) = 0.8\). Now define \(Q^*_{X_1} : X_1 \rightarrow \tau^*_{X_1}\) by \(Q^*_{X_1}(a) = \mu_2; Q^*_{X_1}(b) = \mu_3\). Then, \(Q^*_{X_1}\) is a fuzzy scale and \(Q^*_{X_1} = \{\mu_2, \mu_3\}\). Let \(Q^*_{X_1} = \{0,1,\mu_1,\mu_2\}\) be a fuzzy scalable structure on \(X_1\). Clearly, the triad \((X_1, \tau^*_{X_1}, Q^*_{X_1})\) is a fuzzy scalable structure space.

Let \(X_2=\{u,v\}\) be a non empty set and \(\tau^*_{X_2}\) be the collection of all fuzzy open sets in \(X_2\). Let \(\tau_3=\{0,1,\omega_1,\omega_2\}\) and \(\tau_4=\{0,1,\omega_2\}\) be the fuzzy topologies on \(X_2\) where \(\omega_1, \omega_2, \omega_3 \in \mathbb{T}^X\) are defined as \(\omega_1(u) = 0.3, \omega_1(v) = 0.8\); \(\omega_2(u) = 0.2, \omega_2(v) = 0.3\); \(\omega_3(u) = 0.9, \omega_3(v) = 0.7\). Now, define \(Q^*_{X_2} : X_2 \rightarrow \tau^*_{X_2}\) by \(Q^*_{X_2}(u) = \omega_1; Q^*_{X_2}(v) = \omega_2\). Then, \(Q^*_{X_2}\) is a fuzzy scale and \(Q^*_{X_2} = \{\omega_1, \omega_2\}\). Let \(Q^*_{X_2} = \{0,1,\omega_1,\omega_2\}\) be a fuzzy scalable structure on \(X_2\). Clearly, the triad \((X_2, \tau^*_{X_2}, Q^*_{X_2})\) is a fuzzy scalable structure space.

Let \(f : (X_1, \tau^*_{X_1}, Q^*_{X_1}) \rightarrow (X_2, \tau^*_{X_2}, Q^*_{X_2})\) be a \(FQ^*_X\) continuous function defined by \(f(a) = u\) and \(f(b) = v\). Then the function ‘\(f\)’ is \(FQ^*_X\) b - \(\lambda\) continuous but not \(FQ^*_X\) regular - \(\lambda\) continuous.

Proposition 3.8
Let \((X_1, \tau^*_{X_1}, Q^*_{X_1})\) and \((X_2, \tau^*_{X_2}, Q^*_{X_2})\) be any two fuzzy scalable structure spaces. If a function \(f : (X_1, \tau^*_{X_1}, Q^*_{X_1}) \rightarrow (X_2, \tau^*_{X_2}, Q^*_{X_2})\) is \(FQ^*_X\) \(\lambda\) - continuous then it is \(FQ^*_X\) b - \(\lambda\) continuous.

Remark 3.8
The converse part of the above Proposition 3.8 need not be true as shown in the following example.

Example 3.11
In example 3.10, the function \(f\) is \(FQ^*_X\) b - \(\lambda\) continuous but not \(FQ^*_X\) \(\lambda\) continuous.

Remark 3.9
From the above discussion we have the following implications are true:

![Diagram](Figure: 2 Relationship between \(FQ^*_X\) - pre semi \(\lambda\) continuity and other existing \(FQ^*_X\) continuities)
Bibliography


