APPLICATION OF INTUITIONISTIC FUZZY MATRIX IN DOCUMENT RETRIEVAL SYSTEMS

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Abstract

In this paper, the index and period of an Intuitionistic Fuzzy Matrix (IFM) is determined in terms of its membership parts and its non-membership parts, which leads to the definition of transitive closure of the concept IFM. The knowledge-based Intuitionistic Fuzzy information retrieval method based on Concept Intuitionistic Fuzzy Networks is determined by using the transitive closure of the IFM and Illustrated with suitable examples.

Keywords: Transitive Closure, Fuzzy Matrix, Intuitionistic Fuzzy Matrix, Document Retrieval Systems.

MSC Subject Classification: 15B15, 97R20.

I. INTRODUCTION

Thomason [12], has established that some sufficient conditions for convergence of the powers of a square fuzzy matrix under max-min products. In [5], Kim and Roush have developed that for \(A \in F_n\) is a Fuzzy Matrix of order n, \(A^{k+d} = A^k\) holds for some k, d \(>0\), then the least k \(>0\) such that \(A^{k+d} = A^k\) is called the index of A, the least d \(>0\) such that \(A^{k+d} = A^k\) is called the period of A, that is, every fuzzy matrix has an index and period. Further, for \(A \in F_n\), there exist a positive integer p \(\leq n-1\), such that \(A^p = A^{p+1} = A^{p+2}\) and A \(\sim\) is called the transitive closure of A. The transitive closure of the relation matrices, relevance matrices and document descriptor matrices reveal more accurate results for the system’s user in document retrieval systems based on concept networks and extended fuzzy concept network. A new mechanism based on extended fuzzy concept networks for fuzzy query processing of document retrieval was proposed in [11]. A fuzzy information retrieval method based on fuzzy valued concept networks was discussed in [3,6]. In [13], L.A.Zadeh has been proposed several fuzzy information retrieval methods based on fuzzy set theory. Many researchers have presented techniques to deal with document retrieval using knowledge-based fuzzy information retrieval techniques and the system’s users to perform simple queries, weighted queries, interval queries and weighted interval queries in [1, 2, and 4]. The concept of intuitionistic fuzzy matrices (IFMs) as a generalization of fuzzy matrix was studied and developed by Madhumangal Pal et.al.[10]. For more details on fuzzy matrices one may refer [7]. In our earlier work [8], we have studied on regularity of IFMs. A.R.Meenakshi and M.Kaliraja [9] have been discussed the concept of index and period of an interval valued fuzzy matrices and the fuzzy concept networks of interval valued fuzzy matrices was determined. But, here we discussed about the index, period and
concept fuzzy networks of IFM $A$, as a cartesian product of fuzzy matrices $A_\mu$ and $A_\nu$, where $A_\mu$ represents membership values and $A_\nu$ represents non-membership values of the elements of IFM $A$.

In this paper, the Transitive closure of IFM in Document retrieval systems is computed as a generalization of that of fuzzy document retrieval systems and as extension of an IFM is discussed in [8]. In section 2, the basic definition, notations and required results of an IFM is discussed. In section 3, the index and period on an IFM in terms of that of its membership parts and its non-membership parts is determined, which leads to the definition of transitive closure of the concept IFM. The knowledge-based fuzzy information retrieval method based on concept Intuitionistic fuzzy networks is determined by using the transitive closure of the IFM and Illustrated with suitable examples.

II. PRELIMINARIES

In this section, some basic definitions and notations are given. Let $(IFM)_{mxn}$ denotes the set of all Intuitionistic Fuzzy matrices of order $mxn$. Let $A \in (IFM)_{mxn}$ be represented as Cartesian product of fuzzy matrices. The Cartesian product of any two matrices $A=\{a_{ij}\}_{mxn}$ and $B=\{b_{ij}\}_{mxn}$, denoted as $\langle A, B \rangle$ is defined as the matrix whose $ij^{th}$ entry is the ordered pair $\langle a_{ij}, b_{ij} \rangle$. For $A = \{a_{ij}\}_{mxn} = \langle a_{ij}, a_{ij} \rangle \in F^M_{mxn}$ is defined as the membership part of $A$ and $A_\nu = \{a_{ij}\}_{mxn} = \langle a_{ij}, a_{ij} \rangle \in F^N_{mxn}$ is defined as the non-membership part of $A$. Thus $A$ is the Cartesian product of $A_\mu$ and $A_\nu$ written as $A = \langle A_\mu, A_\nu \rangle$ with $A_\mu \in F^M_{mxn}$, $A_\nu \in F^N_{mxn}$.

The matrix operations on intuitionistic fuzzy matrices is followed and defined in our earlier work [8].

For $A, B \in (IFM)_{mxn}$, if $A = \langle A_\mu, A_\nu \rangle$ and $B = \langle B_\mu, B_\nu \rangle$, then

\( A + B = \langle A_\mu + B_\mu, A_\nu + B_\nu \rangle \) \hspace{1cm} (2.1)

\( AB = \langle A_\mu B_\mu, A_\nu B_\nu \rangle \) \hspace{1cm} (2.2)

$A_\nu \cdot B_\mu$ is the max min product in $F^M_{mxn}$,

$A_\mu \cdot B_\nu$ is the min max product in $F^N_{mxn}$.

The order relation on $(IFM)_{mxn}$ is defined as,

\( A \leq B \iff a_{ij} \leq b_{ij} \) and \( a_{ij} \geq b_{ij} \), for all $i$ and $j \iff A + B = B \) \hspace{1cm} (2.3)

It is well known that [12], every fuzzy matrix has an index and period, that is, for $A \in (IFM)_{n}$, the least $k > 0$ such that $A^{k+d} = A^k$ is called the index of $A$, the least $d > 0$ such that $A^{k+d} = A^k$ is called the period of $A$, these are denoted as $i(A)$ and $p(A)$ respectively.

Lemma 2.1[7]: For $A \in (IFM)_{n}$, if $A^{k+d} = A^k$ for some $k$, $d > 0$, then $k \geq i(A)$ and $p(A)/d$.

III. TRANSITIVE CLOSURE OF AN IFM

In this section, the concept of index and period for an IFM is defined and the relations between the index and period of an IFM $A$ with the indices and periods of membership and non-membership matrices $A_\mu$ and $A_\nu$ is found.

Definition 3.1: For $A \in (IFM)_{n}$, $A^{k+d} = A^k$ holds for some $k$, $d > 0$, then the least $k > 0$ such that $A^{k+d} = A^k$ for some $k$ is called the index of $A$, the least $d > 0$ such that $A^{k+d} = A^k$ for some $d$ is called the period of
A, denoted as \(i(A)\) and \(p(A)\) respectively.

**Theorem 3.2:** For \(A \in \text{IFM}_n\), if \(A = <A_\mu, A_\nu>\) then \(i(A) = \max\{i(A_\mu), i(A_\nu)\}\) and \(p(A) = \text{lcm}\{p(A_\mu), p(A_\nu)\}\).

**Proof:** Let \(i(A) = k\) and \(p(A) = d\), then \(A^{k+d} = A^k\).

Since \(A = <A_\mu, A_\nu>\), by (2.2), \(A_\mu^{k+d}, A_\nu^{k+d} = <A_\mu^k, A_\nu^k>\).

Comparing the corresponding blocks, the result is \(A_\mu^{k+d} = A_\mu^k\) and \(A_\nu^{k+d} = A_\nu^k\).

If \(i(A_\mu) = k_1\), \(i(A_\nu) = k_2\), \(p(A_\mu) = d_1\) and \(p(A_\nu) = d_2\), then by Lemma (2.1), \(k \geq k_1\), \(k \geq k_2\) and both \(p(A_\mu) = d_1\) and \(p(A_\nu) = d_2\) divides \(d\).

Therefore \(i(A) = k \geq \max\{k_1, k_2\} = k'\) and \(\text{lcm}\{d_1, d_2\}/d = d'\). (3.1)

On the other hand, if \(k' = \max\{k_1, k_2\}\), then \(k' \geq k_1\) and \(k_2\) and \(d' = \text{lcm}\{d_1, d_2\}/d\).

By Lemma (2.1), \(A_\mu^{k'+d'} = A_\mu^{k'}\) and \(A_\nu^{k'+d'} = A_\nu^{k'}\). Again by Lemma (2.1), it follows that \(A^{k'+d'} = A^k\). (3.2)

Thus (3.1) and (3.2) yields \(i(A) = \max\{i(A_\mu), i(A_\nu)\}\) and \(p(A) = \text{lcm}\{p(A_\mu), p(A_\nu)\}\). Definition (3.1) and Theorem (3.2) leads to the following definition.

**Definition 3.3:** Let \(M\) be an Intuitionistic Fuzzy matrix of order \(n\). Then there exist an integer \(p \leq n-1\), such that under the composition of IFM, \(M^p = M^{P+1} = M^{P+2} = \ldots\) and \(T = M^p\) is called the transitive closure of an IFM \(M\). Thus the transitive closure of an IFM, \(M = [M_\mu, M_\nu]\) is the Intuitionistic Fuzzy matrix whose membership and non-membership matrices are the transitive closure of \(M_\mu\) and \(M_\nu\), that is, \(M = [M_\mu, M_\nu]\) then \(T = [T_\mu, T_\nu]\) is the transitive closure of \(M\).

\[
T = M^p
\]

\[
[T_\mu, T_\nu] = [M_\mu^p, M_\nu^p] \quad \text{By (2.2)}
\]

\[
T_\mu = M_\mu^p \quad \text{and} \quad T_\nu = M_\nu^p.
\]

**Intuitionistic Fuzzy Concept Networks**

A concept network includes nodes and directed links. Each node represents a concept (or) a document. Each directed link connects concept to concept (or) directs from one concept to a document. Let an interval network with \(n\) concepts \(\{c_1, c_2, \ldots, c_n\}\) and \(m\) documents \(\{d_1, d_2, \ldots, d_m\}\) be considered.

If \(\gamma_{ij} \in [0,1]\) then it indicates that the degree of relevance from concept \(c_i\) to concept \(c_j\) is \(\gamma\), where \(\gamma\in [0,1]\).

If \(\gamma_{ij} \in [0,1]\) then it indicates that the degree of relevance of document \(d_j\) with respect to the concept \(c_i\) is \(\gamma\), where \(\gamma = c_{ij} = c_{ij1} < c_{ij2}, c_{ij3} > \in [0,1]\). (3.3)

Let \(C = (c_{ij})\) denotes the relevant value from the concept \(c_i\) to the concept \(c_j\). The relevant value from concept \(c_i\) to concept \(c_j\) and the relevant value from concept \(c_j\) to concept \(c_k\) are given, that is, \(c_{ij}\) and \(c_{ik}\) are known and \(c_{ik}\) is defined as follows:

\[
c_{ik} = \min\{<c_{ij}, c_{jk}>, (3.4)\}
\]

\[
<c_{ik1}, c_{ik2}> = \min\{<c_{ijk1}, c_{ijk2}, c_{ijk3}>, (3.5)\}
\]

\[
c_{ik} = \min\{c_{ijk1}, c_{ijk2}\} \quad \text{and} \quad c_{ik} = \min\{c_{ijk3}, c_{ijk4}\} \quad \text{By (2.2)}
\]

(3.6)

Similarly, if \(c_{12}, c_{23}, \ldots, c_{(n-1)n}\) are known, then

\[
c_{1n} = \min\{c_{12n}, c_{23n}, \ldots, c_{(n-1)n}\}\]

\[
c_{1m} = \min\{c_{12m}, c_{23m}, \ldots, c_{(n-1)m}\}\]
Definition 3.4: Let \( \{c_1, c_2, \ldots c_n\} \) be a set of \( n \) concepts. A concept Intuitionistic Fuzzy Matrix \( C = (c_{ij}) \) is an \( nxn \) Intuitionistic Fuzzy Matrix, where \( c_{ij} \) is the relevant value from the concept \( c_i \) to the concept \( c_j \) and \( c_{ij} \in [0, 1] \) satisfying the following properties.

(i) Reflexivity: \( c_{ii} = [1, 0] \) for each \( i = 1 \) to \( n \)
\(< c_{ij}, c_{ie} > = [1,0] \) By (3.1)
\( c_{ii} = 1 \) and \( c_{ii} = 0 \) By (2.2), for each \( i = 1 \) to \( n \).

(ii) Non-Symmetric: \( c_{ij} \neq c_{ji} \)
\(< c_{ij}, c_{j} > \neq < c_{ji}, c_{j} > \)
\( c_{ij} \neq c_{ji} \) and \( c_{ij} \neq c_{ji} \)

(iii) Transitivity: \( c_{ik} \geq \max \{< c_{ij}, c_{jk} > \} \)
\( < c_{ik}, c_{ik} > \geq \max \min \{< c_{ij}, c_{ij} > < c_{jk}, c_{jk} > \} \)
\( c_{ik} \geq \max \min \{< c_{ij}, c_{jk} > \} \text{ and } c_{ik} \geq \min \max \{< c_{ij}, c_{jk} > \} \) By (2.3)

Definition 3.5: Let \( \{d_1, d_2, \ldots d_m\} \) be a set of documents and \( \{c_1, c_2, \ldots c_n\} \) be a set of concepts in a concept intuitionistic fuzzy network with \( m \) documents and \( n \) concepts. A document descriptor intuitionistic fuzzy matrix \( D = (d_{ij}) \) is an \( mxn \) matrix, where \( d_{ij} \) is the degree of relevance of document \( d_i \) with respect to the concept \( c_j \).

The document descriptor intuitionistic fuzzy matrix \( D^* = DT \), where \( D \) is the document descriptor of the intuitionistic fuzzy network and \( T \) is the transitive closure of the concept intuitionistic fuzzy matrix. By (2.2), \( D^*_\mu = D_\mu \cdot T_\mu \) and \( D^*_\nu = D_\nu \cdot T_\nu \), indicates the degree of relevance of each document with respect to specific concepts and is used as a basis for similarity measures between queries and documents. Let the above basic concepts in a concept network be illustrated with suitable examples.

Let the concept IFM, Query descriptor, Document descriptor IFM be illustrated and the transitive closure on an IFM be computed using the following examples.

Illustration 3.6: Let a network \( N = (V, E) \) consisting of \( n \) nodes (cities) and \( m \) edges (roads) connecting the cities of a country be considered. If the vehicles on the roads of the network has to be measured at a particular time duration, it is quite impossible to measure the vehicles on a road as a single value because the vehicles at a particular time duration is not fixed, it varies from time to time. In this case, the network (Figure 3.1) is called Intuitionistic Fuzzy networks.

Let us consider a concept Intuitionistic Fuzzy network in Figure 3.1 where \( c_1, c_2, \ldots c_n \) are concepts, \( d_1, d_2, d_3 \) are the documents. If the query descriptor \( Q \) is \( Q = \{(c_3, [1, 0])\} \) where \([1, 0]\) represents the relevant Fuzzy value of the query descriptor \( Q \) with respect to the concept \( c_3 \), then the relevant Fuzzy value of document \( d_2 \) with respect to the concept \( c_3 \) is calculated as follows:
From Figure 3.1 it can be seen that there are three distinct routes from the concept $c_3$ to the document $d_2$.

1. The first route is $c_3[.8, .1] 	o c_1[.5, .5] 	o d_2$. The relevant intuitionistic fuzzy value of the document $d_2$ with respect to concept $c_3$ is calculated by using (3.4) as $\min\{ [.8, .1], [.5, .5] \} = [.5, .1]$.

2. The second route is $c_3[.6, .4] 	o c_5[.7, .2] 	o c_2[.8, .2] 	o d_2$. The relevant intuitionistic fuzzy value of the document $d_2$ with respect to concept $c_3$ is $\min\{ [.6, .4], [.7, .2], [.8, .2] \} = [.6, .2]$.

3. The third route is $c_3[.6, .4] 	o c_5[.2, .8] 	o c_4[.9, .1] 	o d_2$. The relevant intuitionistic fuzzy value of the document $d_2$ with respect to concept $c_3$ is $\min\{ [.6, .4], [.2, .8], [.9, .1] \} = [.2, .1]$.

Then the relevant value of the document $d_2$ with respect to the concept $c_3$ is max $\{ [.5, .1], [.6, .2], [.2, .1] \} = [.6, .2]$. Thus $Q = (c_3, [1, 0]) = [.6, .2]$.

**Example 3.7:** The concept intuitionistic fuzzy matrix $C$ of the network in Figure (3.1) is calculated by using (3.4), (3.5), (3.6).

\[
C = \begin{bmatrix}
1 & <1,0> & <0,0> & <0,0> & <0,0> & <0,0>

1 & <0,0> & <1,0> & <0,0> & <0,0> & <0,0>

1 & <8,1> & <6,2> & <1,0> & <2,4> & <6,4> & <7,2>

1 & <0,0> & <0,0> & <0,0> & <1,0> & <0,0> & <0,0>

1 & <0,0> & <7,2> & <0,0> & <2,8> & <1,0> & <0,0>

1 & <0,0> & <0,0> & <0,0> & <0,0> & <0,0> & <1,0>
\end{bmatrix}
\]

By the Cartesian product representation, $C = \langle C_\mu, C_\nu \rangle$

Where
From the concept fuzzy matrix $C_{\mu}$ and $C_{\nu}$ it is seen that all the diagonal entries are 1 and 0 respectively. That is, $c_{ii}^{\mu} = 1$ for $i = 1$ to 6. Here $c_{23}^{\mu} = 0$ but $c_{32}^{\mu} = 0.6$ and therefore $c_{ij}^{\mu} \neq c_{ji}^{\mu}$ and $c_{ii}^{\nu} = 0$ for $i = 1$ to 6. Here $c_{23}^{\nu} = 0$ but $c_{32}^{\nu} = 0.2$ and therefore $c_{ij}^{\nu} \neq c_{ji}^{\nu}$

Hence, by (2.2), $c_{ii} = \begin{bmatrix} 1 & 0 \end{bmatrix}$ for $i = 1$ to 6. Here $c_{23} = \begin{bmatrix} 0 & 0 \end{bmatrix}$ but $c_{32} = \begin{bmatrix} .6 & .2 \end{bmatrix}$ and therefore $c_{ij} \neq c_{ji}$. Hence $C$ is not symmetric.

Example 3.8: The document descriptor intuitionistic fuzzy matrix $D$ for the concept intuitionistic fuzzy network in Figure (3.1) is computed as in Example (3.6).

Then '0' entries in the $D_{\mu}$ and $D_{\nu}$ indicate that the corresponding concepts are not relevant or can be neglected with respect to the particular document. For instance the concepts $c_2, c_4, c_5$ and $c_6$ are not relevant for the document $d_1$ in $D_{\mu}$ and $D_{\nu}$. To get the implicit relevant values of each document with respect to specific concepts, let the transitive closure of the concept matrix given in Example (3.7)
be computed for the concept intuitionistic fuzzy network in Figure (3.1).

\[ C^2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0.6 & 1 & 2 & 6 & 7 & 0.6 \\
0 & 0 & 1 & 0 & 0 \\
0.7 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

\[ C^2 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0.6 & 1 & 2 & 6 & 7 & 0.6 \\
0 & 0 & 1 & 0 & 0 \\
0.7 & 0 & 2 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

Similarly \( C^3 = C \). This implies that \( C = C^3 \). Therefore \( C \) itself is the transitive closure of the concept intuitionistic fuzzy matrix. Since \( C = T \), \( T_\mu = C_\mu \) and \( T_\nu = C_\nu \).

\[ D_\mu^* = D_\mu \cdot T_\mu = \begin{bmatrix}
7 & 6 & 1.2 & 6 & .7 \\
.6 & .8 & .6 & 9 & .6 & .7 \\
7 & 6 & 1.2 & \# & .8 \\
\end{bmatrix}
\]

and

\[ D_\nu^* = D_\nu \cdot T_\nu = \begin{bmatrix}
.1 & .1 & .1 & .1 & .1 \\
0 & .1 & 0 & .1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Therefore

\[ D^* = \langle D_\mu \cdot T_\mu, D_\nu \cdot T_\nu \rangle = \begin{bmatrix}
<.7, 1> & <.6, 1> & <.7, 1> & <.2, 1> & <.6, 1> & <.7, 1> \\
<.6, 0> & <.8, 1> & <.6, 0> & <.9, 1> & <.6, 0> & <.7, 0> \\
<.7, 0> & <.6, 0> & <.7, 0> & <.2, 0> & <.6, 0> & <.8, 0> \\
\end{bmatrix}
\]

The document descriptor intuitionistic fuzzy matrix \( D^* \) gives the implicit values of each document more accurately. For instance [0, 0] entries in the first row of the document descriptor IFM in Example (3.8) are improved as [.6, .1], [.2, .1], [.6, .1] and [.7, .1] respectively, that is, the concepts \( c_2 \), \( c_4 \), \( c_5 \) and \( c_6 \) cannot be neglected for the document \( d_1 \). Thus the document descriptor intuitionistic fuzzy matrix \( D^* \) obtained by using the transitive closure \( \cdot T \) reveals more accurate results for the user.

IV. CONCLUSION

The index, period and the transitive closure of an IFM is determined, in terms of its membership and non-membership fuzzy matrices. These concepts are illustrated with the help of a simple intuitionistic fuzzy document retrieval method as a generalization of the results found in [1, 2, 3 and 13].
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