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Generalization Of Selfmaps And Contraction Mapping Principle In D-Metric Space.

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ABSTRACT

Large number of fixed point results for selfmappings satisfying various types of contractive inequalities which in [5, 9 ,12]. In this paper, theorems on selfmaps and some fixed point theorems are proved in D-metric spaces. The Generalization of contraction mapping principle in D-metric space which include some fixed point results in U.P.Dolhare [1], B.C. Dhage, U.P.Dolhare and Adrian Petrusel [3] ,Rhoades [12] in D-metric spaces as special cases.

Key words: D-Metric space, fixed point etc.,(2000) Mathematics subject classification:47H10, 54H25

I. INTRODUCTION

The study of fixed points of Selfmappings satisfying contractive conditions which is one of the research activity. Metric fixed point theory used to Banach fixed point theorem (1922). Fixed point theorems are applied in various fields of science. The theory started with the generalization of the Banach fixed point theorem using contraction and non expansive mappings Pant [15], Fisher B [6], Rhoades B.E.[8], Cric [12]. worked 0n these mapping. Dhage, Dolhare U.P. and Andrian Petrusel [2] used non-selfmaps to obtain some fixed point theorems in metric space. M.S.Khan[9] proved some interesting results for multivalued continuous mappings. We also generalized the results of Chtterjee s.[14] to expansive selfmaps and non-selfmaps.

II. EXPANSION MAPPINGS WITH FIXED POINTS

***Rhoades** proved the folling theorem:*

<u>Theorem 2.1</u> :If f is a selfmap of complete metric space (x,d), f is onto, and there is a constant $\alpha > 1$ such that

$$d(f_x, f_y) \geq \alpha d(x,y), \text{ for all } x, y \in X \tag{2.1}$$

then f has a unique fixed point.

Pant .R,P.[16] generalizing this theorem by considered the following condition.

$$d(f_x, f_y) > \min\{d(x,y), d(x , f_x), d(y , f_y)\} \text{ for all } x, y \in X, \ x \neq y. \tag{2.2}$$

He proved each continuous selfmap f of a compact metric space satisfying the above condition has a fixed point. **Dhage,Dolhare u.p and Ntouyas s.k**.[4] proved some interesting results for multivalued continuous mappings. We investigate the fixed points of continuous mappings , expansive mappings for non-selfmaps and obtain some results. **M.S.Khan** [9] used the square root condition for h < 1 to obtain a common fixed point of two continuous self mappings S and T

$$d(S_x,T_y) \leq h\{ d(x,S_x). d(y, T_y) \}^{½} \qquad \text{for } h < 1 \tag{2.3}$$

he used the following expansive condition to obtain an identity mapping :

$$d(T_x, T_y) \geq \{d(x, T_x).d(y, T_y)\}^{1/2} \qquad \text{for all } x, y \in X$$

if we replace the contractive condition (2.3) with the similar expansive condition :

$$d(S_x, T_z) \geq h\{d(x, S_x).\, d(z, T_z)\}^{1/2} \qquad \text{for } h>1 \qquad (2.4)$$

This condition which is useful for to find the fixed point.

III. D-CONTRACTION MAPPING PRINCIPLE:

One of the most fundamental fixed point theorems for Contraction mappings in complete metric space is proved by Polish Mathematician **Stefan Banach** in year 1922 which as follows.

Theorem 3.1 : *Let f be a self mapping of a complete metric space X satisfying*

$$x, y \in X \text{ and } 0 \leq d(f_x, f_y) \leq \alpha\, d(x, y) \text{ for all } \alpha < 1$$

then f has unique fixed point x and the sequence* $\{f_1^n(x)\} \in X$ *of the successive iterations of* $f_1^n(x) \in X$ *converges to x**

Similarly the fundamental fixed point theorems for contraction as will as contractive mappings in D-Metric spaces are proved by **Dolhare and Dhage** [5] which as follows.

Theorem 3.2 : Let f be self mapping of a complete and bounded D- metric space X satisfying

$\varrho(f_x, f_y, f_z) \leq \alpha\, \varrho(x, y, z)$ **for all** $x, y, z \in X$ **and** $0 \leq \alpha < 1$

then f has a unique fixed point x* and the sequence $\{f^n{}_x\} \in X$ of the successive iterations $f_x \in X$ converges to x*.

Also **Caccioppoli** [10] proved the following fixed point theorem in D-Metric spaces.

Theorem 3.4 : Let f be a selfmap of a complete metric space X satisfying

$d(f^n{}_x, f^n{}_y) \leq a_n\, d(x, y)$ *for all* $x, y \in X$ *where* $a_n > 0$, $n \in N$ (3.1) and are independent of x and y if $\sum_{n=1}^{\infty} an < \infty$ then f has a unique fixed point.

Dhage ,Dolhare and Ntouyas S.K. [4] give the following form of well known D-Contraction mapping principle.

Theorem 3.5 : Let f be a selfmaps of f-orbitally complete D-metric space X satisfying

$\varrho(f_x, f_y, f_z) \leq \lambda\, \varrho(x, y, z)$ for all $x, y, z \in X$, where $0 \leq \lambda < 1$ then f has a unique fixed point.

IV. GENERALIZATION OF CONTRACTION MAPPING PRINCIPLE IN D-METRIC

SPACE :

The study of the results concerning nonlinear self mappings satisfying a special type of contraction condition on a D-metric space are obtained. The study of the nonlinear mapping f on a D-metric space X into itself satisfying the contractive condition of the form

$$\varrho(f_x, f_y, f_z) \leq \emptyset[\max\{\varrho(x, y, z), \varrho(x, f_x, f_y), \varrho(x, f_x, f_z), \varrho(y, f_y, f_x), \varrho(y, f_y, f_z), \varrho(y, f_z, f_y),$$
$$\varrho(z, f_z, f_x),)\}] \qquad \text{for all } x, y, z \in X$$

where $\emptyset: R^+ \rightarrow R^+$ is a continuous non decreasing function satisfying $\emptyset(t) < t$, $t>0$ and $\sum_{n=1}^{\infty} \emptyset^n(t) < \infty$ for each $t \in [0,\infty)$ and the existence of a unique fixed point is proved under the condition of completeness and boundedness of X .

In 1922 Banach first proved his well-known contraction mapping principle for the selfmappings in ordinary metric spaces satisfying certain contraction condition.In 1968 Kannan give a new tern to Banach fixed point theorem and discovered a new class of contraction mappings.

Banach contraction mapping principle are several extension and generalization of the above contraction principle in literature see **Rhoades** [11], **Ciric[**12].

Theorem 4.1 : Ciric[12] : Let f be a selfmap of a complete metric space X
$d(f_x, f_y) \leq C \max\{d(x, y), d(x, f_x), d(y, f_y), d(x, f_y), d(y, f_x)\}$ *for all* $x, y \in X$, *where* $0 \leq C \leq 1$ *then f has a unique fixed point.*

Rhodes [11] Characterized general Contraction mappings by the inequality

$$d(f_x, f_y) \leq C \max\{ d(x, y), d(x, f_x), d(y, f_y), [d(x, f_y) + d(y, f_x)/2]\}$$

then f has unique for all x, y ∈ X, where $0 \leq C \leq 1$ then f has unique fixed point.

Dolhare [1] and **Dhage and Rhoades**[13] shows that fixed point theorem in D-metric space are natural generalization of the fixed point theorems in metric spaces.

V. CONTRACTION MAPPINGS IN D-METRIC SPACES :

The basic contraction mapping principle in D-Metric spaces developed by Dhage [3] as follows

Definition : Let X be a D-metric space and $f: X \to X$. Let $O_f(x)$ denote an f orbit of f at a point $x \in X$ defined by $O_f(x) = \{ x, f_x, f^2{}_x ,------\}$ D-metric space is called f-orbitally complete if every D-Cauchy sequence in $O_f(x)$ converges to a point in X

Similarly X is called f-orbitally bounded if $(O_f(x)) < \infty$ for each $x \in X$

Theorem 5.1 [Basic contraction mapping principle]
Let X be a D-metric space and Let $f: X \times X \to X$ a mapping satisfying
$\varrho(f_x, f_y, f_z) \leq \lambda(\max\{\varrho(x, y, z), \varrho(x, f_x, z), \varrho(x, f_y, z), \varrho(y, f_x, z), \varrho(f_x, f_y, f_z)\})$ $\forall$ x, y, z ∈X, where $0 \leq \lambda \leq 1$ further if X is f-orbitally bounded and f-orbitally complete, then f has a unique fixed point.

Lemma 5.1 : If $\emptyset \in \Phi$ then $\emptyset^n(0) = 0$ for each $n \in N$ and $\lim_n \emptyset^n(t) = 0$ for all $t > 0$
By generalizing contraction mapping principle Dolhare [7] we prove our main result

VI. MAIN RESULT

The natural generalization of contraction mapping principle in D-metric is as follows.

Theorem 6.1 : Let X be a D-metric space and let $f : X \to X$ be a mapping satisfying

$$\varrho(f_x, f_y, f_z) \leq \emptyset \max\{ \varrho(x, y, z), \varrho(x, f_x, f_y), \varrho(x, f_x, f_z), \varrho(y, f_y, f_z), \varrho(y, f_y, f_x), \varrho(z, f_z, f_x), \varrho(z, f_z, f_y)\}. \quad (6.1)$$

Proof: Let x ∈ X be arbitrary and define $\{x_n\} \subset X$ by $x_0 = x$, $x_{n-1} = fx_n$, $n \geq 0$ if $x_r = x_{n-1}$ for some r ∈ N, then $U = x_r$ is a fixed point of f .Therefore we assume that $x_n \neq x_{n-1}$ for each n ∈ N.
We show that $\{x_n\}$ is D-Cauchy.
By using the condition $x = x_0$, $y = x_1$ and $z = x_{m-1}$, $m > 1$ in (6.1) we obtain

$$\varrho(x_1, x_2, x_m) = \varrho(fx_0, fx_1, fx_{m-1})$$
$$\leq \emptyset(\max\{ \varrho(x_0,x_1,x_{m-1}), \varrho(x_0,fx_0,fx_1), \varrho(x_0,fx_0,fx_{m-1}), \varrho(x_1,fx_1,fx_{m-1}), \varrho(x_1,fx_1,fx_0), \varrho(x_{m-1},fx_{m-1},fx_0), \varrho(x_{m-1},fx_{m-1},fx_1)\})$$
$$= \emptyset(\max(\varrho(x_0, x_1, x_{m-1}), \varrho(x_0, x_1, x_1), \varrho(x_0, x_1, x_m), \varrho(x_1, x_2, x_m), \varrho(x_1, x_2, x_1), \varrho(x_{m-1}, x_m, x_1), \varrho(x_{m-1}, x_m, x_2)\})$$

$$\leq \emptyset \begin{pmatrix} max\varrho(xa, xb, xc) \\ 0 \leq a \leq n+1 \\ 1 \leq b \leq m-1 \\ 2 \leq c \leq m \end{pmatrix}$$

$\leq \emptyset(k)$

Again getting $x = x_1$, $y = y_2$, $z = x_{m-1}$, $m > 2$ in 6.1 yields

$\varrho(x_2,x_3,x_m) = \varrho(fx_1, fx_2, fx_{m-1})$

$\leq \emptyset$ (max { $\varrho(x_1,x_2,x_{m-1})$, $\varrho(x_1,x_2,x_3)$, $\varrho(x_1,x_2,x_m)$ $\varrho(x_2,x_3,x_m)$,$\varrho(x_2,x_3,x_2)$, $\varrho(x_{m-1},x_m,x_2)$, $\varrho(x_{m-1},x_m,x_3)$})

$$\leq \emptyset^2 \begin{pmatrix} \max\varrho(xa, xb, xc) \\ 0 \leq a \leq 3 \\ 1 \leq b \leq m-1 \\ 2 \leq c \leq m \end{pmatrix}$$

$\leq \emptyset^2(k)$

By induction

$$\varrho(x_n,x_{n-1},x_m) \leq \emptyset^n \begin{pmatrix} \max\varrho(xa, xb, xc) \\ 0 \leq a \leq n+1 \\ 1 \leq b \leq m-1 \\ 2 \leq c \leq m \end{pmatrix}$$

$\leq \emptyset^n(k)$ for all $m > n \in N$

Now an application of theorem 5.1 yield that $\{x_n\}$ is D-Cauchy X being a complete D-metric space there is a point $u \in X$ such that $\lim_n xn = u$. We show that u is a fixed point of f

Now $(u,u,fn) = \lim_n \varrho(x_{n+1},x_{n+1},f_u)$

$= \lim_n (fx_n, fx_n, f_u)$

$\leq \lim_n \emptyset(\max\{\varrho(x_n, x_n, u), \varrho(x_n, x_{n-1}, x_{n-1}), \varrho(x_n, x_{n-1}, f_u), \varrho(x_n, x_{n-1}, f_u), \varrho(x_n, x_{n-1}, x_{n-1}), \varrho(u, f_u, x_{n-1}), \varrho(u, f_u, x_{n-1})\})$

$= \emptyset(\max\{0,0, \varrho(u,u,f_u)\})$

$= \emptyset(\varrho(u,u,f))$

Which is possible only when $u = f_u$ thus f has unique fixed point

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